The fetters of the sib: Weber meets Darwin

Ingela Alger, Jörgen Weibull

ECOLE POLYTECHNIQUE

CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

THE FETTERS OF THE SIB: WEBER MEETS DARWIN

Ingela ALGER
Jörgen W. WEIBULL

September 2008

Cahier n° 2008-27

DEPARTEMENT D'ECONOMIE
Route de Saclay
91128 PALAISEAU CEDEX
(33) 1 69333033
http://www.enseignement.polytechnique.fr/economie/
mailto:chantal.poujouly@polytechnique.edu

THE FETTERS OF THE SIB: WEBER MEETS DARWIN

Ingela ALGER[1]
Jörgen W. WEIBULL [2]

September 2008

Cahier n° 2008-27

Abstract: We analyze how family ties affect incentives, with focus on the strategic interaction between a pair of mutually altruistic siblings. Each sibling exerts effort to produce output under uncertainty and the siblings may transfer output to each other. With equally altruistic siblings, their equilibrium effort is non-monotonic in the common degree of altruism and depends on the harshness of the environment. We define a notion of local evolutionary robustness of degrees of sibling altruism, and show that this degree is less than one half, the kinship relatedness factor. By way of numerical simulations we show that family ties are weaker in harsher environments.

Key Words : altruism, family ties, Hamilton's rule, free-riding, evolutionary robustness.

JEL Classification : D02, D13

[1] Carleton University
[2] Stockholm School of Economics, Department of Economics, Ecole Polytechnique

The Fetters of the Sib: Weber Meets Darwin

Ingela Alger and Jörgen W. Weibull*
Carleton University and Stockholm School of Economics

September 22, 2008

Abstract

We analyze how family ties affect incentives, with focus on the strategic interaction between a pair of mutually altruistic siblings. Each sibling exerts effort to produce output under uncertainty and the siblings may transfer output to each other. With equally altruistic siblings, their equilibrium effort is non-monotonic in the common degree of altruism and depends on the harshness of the environment. We define a notion of local evolutionary robustness of degrees of sibling altruism, and show that this degree is less than one half, the kinship relatedness factor. By way of numerical simulations we show that family ties are weaker in harsher environments.

Keywords: altruism, family ties, Hamilton's rule, free-riding, evolutionary robustness.

JEL codes: D02, D13

---

*We are grateful to Daron Acemoglu, Philippe Aghion, Richard Arnott, Ted Bergstrom, Margherita Bottero, Don Cox, Avinash Dixit, Justina Fischer, Rachel Kranton, Olof Leimar, Karen Norberg, Marcus Salomonsson, and Yannick Viossat for helpful comments and discussions. We also thank audiences at Boston College, Boston University, the Canadian Economics Association, Carleton University, the Econometric Society, the European Economics Association, HEC Montréal, the PET Workshop in Singapore, SUNY Binghamton, Université de Cergy-Pontoise, Université Laval, University of Guelph, University of Hong Kong, and Universität Zürich for feedback. The usual disclaimer applies. Both authors thank the Knut and Alice Wallenberg Research Foundation for financial support, and Ingela Alger thanks the Stockholm School of Economics for its hospitality.

> [B]etween the frozen pole of egoism and the tropical expanse of utilitarianism [there is] (...) the position of one for whom in a calm moment his neighbour's utility compared with his own neither counts for nothing, nor 'counts for one', but counts for a fraction. (F.Y. Edgeworth, *Mathematical Psychics,* 1881, Appendix IV)

1 Introduction

As much as economists cherish the assumption that individuals are selfish, altruistic behavior, such as gift giving, material assistance, and cooperation in social-dilemma-like situations, is common. While such behavior may arise as an equilibrium outcome in an indefinitely repeated interaction between selfish individuals many economists, including Edgeworth (1881) and Becker (1974), have theorized that altruism exists. Most people would probably also find, by introspection, that they are willing to sometimes help others, even with no prospect of future rewards. Not surprisingly, an extensive theoretical and empirical literature has developed to investigate how altruism affects economic outcomes and how altruistic behaviors are sustained.[1] In this paper we shed new light on both questions, with a focus on family ties.

Numerous empirical studies show that private transfers are more common within the family than between unrelated households,[2] and that such transfers appear to function as a risk-sharing device.[3] Intuition suggests that high levels of informal risk sharing within the family are desirable. However, several researchers have offered rather negative views of the family. Thus, Banfield (1958) thought that the "amoral familism" that he observed in certain parts of Italy was an impediment to economic development. In a similar spirit, Max Weber (1951) thought that "the great achievement of [...] the ethical and ascetic sects of Protestantism was to shatter the fetters of the sib [the extended family]. These religions established [...] a common ethical way of life in opposition to the community of blood, even

[1] For a recent collection of surveys see Kolm and Ythier (2006)

[2] See Cox and Fafchamps (2008), and Fafchamps and Lund (2003).

[3] Cox, Galasso and Jimenez (2006) show that the average income of donor households exceeds that of recipient households. Fafchamps and Lund (2003) find that shocks affect transfers between Filipino rural households. Using data from Thailand, Miller and Paulson (1999) show that remittances respond to shocks to regional rainfall.

to a large extent in opposition to the family. (p.237)" In his view, a strong sense of solidarity among members of the extended family, coupled with a hostile attitude towards strangers, promotes a culture where nepotism may thrive and counter the efficient development of markets.

Motivated by evidence that family ties vary in strength across cultures,[4] we here pursue the line of thought suggested by Weber, by theoretically analyzing the effects of family ties on risk-sharing and incentives. We address several questions, including: If family members with higher earnings give transfers to those with lower incomes (and are willing and expected to do so), what is the effect of such family ties on incentives to exert productive effort or make productive investments? What is the most efficient level of informal risk sharing, if any? We are able to shed new light on these classical issues by allowing for mutual altruism and an endogenous risk-reducing effort, where the literature has focused either on models with one-sided altruism, or on models with mutual altruism but without risk.

Furthermore, inspired by observations by Weber and others (see below) that family ties may have grown weaker in northwestern Europe prior to the industrial revolution, we ask whether the incentive effect of family ties (in a society consisting mainly of subsistence farmers) depends on the exogenously given environment.[5] If so, may this have contributed to the development of relatively weak family ties in certain parts of the world? We formally address this question by determining evolutionarily robust degrees of altruism.

Our model is simple, but, we believe, canonical: two risk-averse siblings each choose a costly risk-reducing action, "effort," that determines the probability distribution over output levels. Once both siblings' outputs have been realized, each sibling chooses whether to share some of his or her output with the other.[6] We model the motive for intra-family transfers as altruism, modelled in the usual way as a positive weight placed on other family members'

[4] Alesina and Giuliano (2007) use the World Values Survey to establish that family ties vary in strength among different countries. Evidence based on rates of cohabitation between parents and their adult children shows that such cohabitation is (on average) viewed as an inferior good in the U.S. (Rosenzweig and Wolpin, 1993), but as a normal good in Italy (Manacorda and Moretti, 2006).

[5] In a companion sequel paper, Alger and Weibull (2008), we analyze these questions in a setting in which family transfers are socially coerced rather than, as here, voluntary, and there we also compare the outcomes with those in perfectly competitive insurance markets.

[6] Other researchers have taken the risk as given and focussed on the enforceability of transfers within families; see, e.g., Coate and Ravallion (1993), Foster and Rosenzweig (2001), Genicot and Ray (2003), and Bramoullé and Kranton (2007).

welfare. This particular game has not been studied before. Most of the literature on altruism, starting with Becker (1974), assumes one-sided altruism (see also, e.g., Bruce and Waldman, 1990, Chami, 1998, and Lindbeck and Nyberg, 2006). In models with two-sided altruism, typically only one of the players chooses an effort (see Laferrère and Wolff, 2006, for a recent survey), or there is no risk (Lindbeck and Weibull, 1988, and Chen and Woolley, 2001).

In the case of equally altruistic siblings, an increase in the common level of altruism leads to larger transfers, and thus a stronger free-rider effect on effort, but also to a stronger empathy-effect on effort, by which we mean the desire to be able to help one's sibling if need be. Which effect dominates? It turns out that both effects are absent when the common degree of altruism is low, that the free-rider effect outweighs the empathy effect when the common degree of altruism is of intermediate strength, and that the opposite holds when the common degree of altruism is strong. Despite the non-monotonicity of effort, with respect to the common degree of altruism, siblings fare best, in terms of their expected material utility — utility from consumption and effort — when they are fully altruistic towards each other (attaching the same weight to the other's material utility as to their own). In particular, their expected material utility is higher than if they had been completely selfish.

Although full altruism would lead to the (*ex ante* expected) Pareto efficient outcome, full altruism is not what we observe in reality.[7] What level of intra-family altruism should one expect, from first principles? Here we follow in the footsteps of early proponents of evolutionary theory, including Darwin, who were puzzled by the occurrence of altruism in nature: how can a behavior or trait whereby the individual gives up resources for the benefit of others survive? Since then, biologists have developed theories of altruism, such as kinship altruism (Haldane, 1955, and Hamilton, 1964a,b), reciprocal altruism (Trivers, 1971), and multilevel selection (Sober and Wilson, 1998). Our approach is closest to that of the British biologist William Hamilton (1964a,b), and in a sense we generalize the so-called Hamilton's rule, much along the same line as proposed by Bergstrom (1995). Hamilton's model, "which is particularly adapted to deal with interactions between relatives of the same generation" (Hamilton, 1964a, p.2), predicts that evolutionary forces will select for a degree of altruism of approximately 1/2 between siblings. According to Hamilton "This means that for a hereditary tendency to perform an action [which is detrimental to individual fitness] of this

[7]The large empirical literature on intra-family transfers was recently reviewed by Cox and Fafchamps (2008). Cox, Hansen, and Jimenez (2004), and Maitra and Ray (2003) find fairly strong evidence that transfers are driven by altruistic motives for low-income households, although there is no evidence that such altruism would be anywhere near full altruism.

kind to evolve the benefit to a sib must average at least twice the loss to the individual." (op. cit., p.16). Such an action would be beneficial to "inclusive fitness," a notion introduced by Hamilton in this article. This has become known as Hamilton's rule, and can be summarized as the condition that the action in question will be taken if and only if $rb > c$, where c is the reduction of the actor's fitness, b is the increase in the recipient's fitness, and r is *Wright's coefficient of relationship*, a coefficient that is 1/2 between siblings (Wright, 1922).

When postulating his rule, Hamilton did not consider strategic aspects of the interaction between kin. Ted Bergstrom (1995, 2003) enriched Hamilton's kinship selection theory by allowing for precisely such aspects. Inspired by Bergstrom's (1995, 2003) approach, we develop a notion of *local evolutionary robustness* and apply this to the above-mentioned pairwise sibling interaction. We show that neither complete selfishness (no concern for one's sibling), nor full altruism (equal concern for one's sibling as for oneself) is locally evolutionarily robust in any environment. In the light of *Hamilton's rule*, at first sight one might conjecture the locally evolutionarily robust degree of altruism to equal one-half, the coefficient of relationship between the siblings. This would indeed be true in our model, had effort levels been exogeneously fixed. However, we show that the strategic aspects that endogeneously determine the siblings' efforts pushes the locally evolutionarily robust degree of altruism down, to a level below 1/2, and that it depends on the harshness of the environment. An individual with sibling altruism 1/2 can be exploited by a more selfish "mutant" sibling, and this tendency is stronger in harsher climates. As a result, family ties should be expected to be weaker in harsher environments (or climates) than in milder ones. This theoretical finding seems to be consistent with empirical observations that family ties grew weaker in the harsh northwestern Europe prior to the industrial revolution (see Section 6). Returning to Weber's observation that Protestantism "shattered the fetters of the sib:" if altruism is lower in the (usually harsher) climates of Protestant countries, then Protestantism need not be the *cause* for weaker family ties, but the *result* of harsher climates — chosen as a moral code for the looser family ties that typically prevail in harsh climates. Indeed, some historians share the view that early Protestantism arose predominantly in areas where traditional social norms and social expectations were at odds with the rules imposed by Rome (see e.g. Ozment, 1974, 1992).

We are not aware of any work leading to these predictions. The closest is probably Bergstrom (1995), mentioned above, who notes that a population consisting of individuals who discount the fitness benefit bestowed on their siblings by one half would not resist an invasion by mutants with a different discount factor (degree of altruism). Eshel and Shaked

(2001) develop a model of partnerships, in which individuals may protect each other against hazards in order to increase the likelihood of having someone around to help back in the future. When considering interactions between relatives, however, Eshel and Shaked assume that people use the genetic kinship factor, in accordance with Hamilton's rule.

Our base model is close to that by Arnott and Stiglitz (1991). They model "family insurance" as transfers within pairs of ex ante identical individuals and they allow for an endogenous, risk-reducing effort taken by these individuals. However, whereas in our model transfers within the family are driven by altruism, in their model family transfers are the outcome of a joint agreement. In particular, if family members can observe each other's effort, the joint agreement in their model specifies that total family income should always be split equally and (in the case of observable effort) the agreement specifies the effort to be taken. Mathematically, this is equivalent to the special case in our model of maximal family altruism (when members attach the same utility weight to other's welfare as to their own). Moreover, they address a different question. They ask whether, in the presence of insurance markets, supplemental informal insurance within the family improves welfare.

The topic we address here is also related to that in Lindbeck and Nyberg (2006), who analyze altruistic parents' incentive to instill a work norm in their children. The incentive stems from parents' inability to commit not to help their children if in financial need. If the children feel a strong social norm to work hard, then this reduces the risk that the children will be in need, which is good for the altruistic parents. They focus on parent-child interactions and do not carry out an evolutionary stability analysis.

The remainder of the paper is organized as follows. In the next section we set up the model, beginning with the case of a selfish atomistic individual and then introducing family ties in terms of a two-stage game between two mutually altruistic siblings. In section 3 we characterize equilibria, and we conduct comparative-statics analyses of the equilibrium outcome. In Section 4 we develop a notion of evolutionary robustness of family ties and apply this to our model. Section 5 briefly discusses evidence on family ties, and Section 6 concludes. All mathematical proofs have been relegated to an appendix.

2 The model

2.1 Atomistic and selfish individuals

Consider an individual who feels no wish or social pressure to help others, living in an environment where insurance is not available. The individual chooses an *effort* level $x \in \mathbb{R}_+$ that determines the probability distribution over the possible returns, or *output* levels. The output is either high, y^H, or low, $y^L = \lambda y^H$, where $\lambda < 1$ is the factor by which output is reduced in the "bad" outcome. As such, λ represents *output variability*. With probability $\kappa \in [0, 1)$ an exogenous hazard, such as a natural catastrophe, leads the output to being low; the parameter κ may also be interpreted as institutional quality, e.g., the probability with which private property will be confiscated. When this hazard does not strike, the output is high with probability p and low with probability $1-p$. The probability $p \in [0, 1]$ for the high output level (when the exogenous hazard does not strike) is increasing (at a decreasing rate) in the individual's effort, $p = \varphi(\theta x)$, where $\varphi : \mathbb{R}_+ \to [0, 1)$ is continuously differentiable with $\varphi(0) = 0$, $\varphi' > 0$, $\varphi'' < 0$ and $\varphi(x) \to 1$ as $x \to +\infty$. The parameter $\theta > 0$ represents the ease with which effort increases the probability of the high output: a higher θ implies that the effort required to achieve a given success probability p is smaller: $x = \varphi^{-1}(p)/\theta$. A higher θ may thus represent an easier environment, more skillful individuals and/or technological progress. We will refer to θ as the *effort return* parameter. Note that, by assumption, $p = \varphi(\theta x) < 1$ for all θ and x. In other words: it is impossible for any inidividual in any environment to obtain the high output level for sure.

Since the low output level is achieved without any effort, this is the output that nature provides "for free." By contrast, the high output level is the best that can be achieved with effort. In most of our comparative statics analyses, we will keep the high output level, $y^H = Y > 0$, fixed while the three other "environmental" parameters, λ, θ, and κ, vary. We will usually refer to the triplet $(\lambda, \theta, \kappa)$ as *the environment*. We will say that an environment $(\lambda', \theta', \kappa')$ is *harsher* than another environment $(\lambda, \theta, \kappa)$ if the low output is lower ($\lambda' \leq \lambda$), the marginal return to effort is smaller ($\theta' \leq \theta$), and/or the probability of the exogenous hazard is higher ($\kappa' \geq \kappa$), with at least one strict inequality.

In a given environment, an effort level $x \geq 0$ results in the expected utility

$$(1-\kappa)\,\varphi(\theta x)\,u(Y) + [(1-\kappa)(1-\varphi(\theta x)) + \kappa]\,u(\lambda Y) - v(x), \tag{1}$$

where $u(y)$ is the utility from consuming an amount $y > 0$ and $v(x)$ the disutility (or cost)

of exerting effort $x \geq 0$. We assume that both u and v are twice differentiable with, $u' > 0$, $u'' < 0$, $v' \geq 0$, $v'' > 0$ and $v'(x) = 0$ if and only if $x = 0$.

Alternatively, if the individual chooses his or her success probability p, at a cost or disutility $\psi(p)$, the expected utility can be written as

$$(1-\kappa)\, pu(Y) + [(1-\kappa)(1-p) + \kappa]\, u(\lambda Y) - \psi(p), \tag{2}$$

where u is defined as above and ψ can be derived from v and φ as follows: $\psi(p) = v(\varphi^{-1}(p)/\theta)$. The previous assumptions on v and φ imply that, for any given $\theta > 0$ the disutility of maintaining a success probability p is increasing and strictly convex in p: $\psi' \geq 0$ and $\psi'' > 0$, with $\psi'(p) = 0$ if and only if $p = 0$, and $\psi'(p) \to +\infty$ as $p \to 1$. The optimal success probability $p \in (0,1)$ is uniquely determined by the first-order condition

$$\psi'(p) = (1-\kappa)[u(Y) - u(\lambda Y)], \tag{3}$$

which simply requires that the marginal disutility of increasing the success probability should equal the marginal benefit thereof. We note that the success probability defined by (3) is higher the higher is the variability λ of the environment, and, when translating the model specification back to the effort-based model, the higher is the marginal return to effort θ. In the sequel we will use this model verions and let x^0, p^0, y^0, and V^0 denote the effort, success probability, expected income, and expected utility of an atomistic and selfish individual.

2.2 Individuals with family ties

Now assume that these individuals still work individually but belong to families in which the members have altruistic feelings towards each other. In case of unequal individual output levels between siblings, those who obtained higher output may want to share some of their output with less fortunate members.[8] More precisely, assume now that there are two siblings, A and B, who interact over two periods, along the lines of the model in the preceding section. Thus, in the first period, both siblings simultaneously choose their individual success probabilities. Let $\mathbf{p} = (p_A, p_B)$ be the success-probability vector. The output y_i of each individual $i = A, B$ is realized at the end of the first period. The exogenous hazard, that occurs with probability κ, is taken to be a common shock that brings both siblings' outputs

[8] As will be shown below, an alternative interpretation is that family members are selfish but can sign contracts on conditional transfers.

to the low level, λY. The probability for the output pair $\left(y^H, y^H\right)$ is thus $(1-\kappa)\, p_A p_B$, that for $\left(y^H, y^L\right)$ is $(1-\kappa)\, p_A\,(1-p_B)$, that for $\left(y^L, y^H\right)$ similarly is $(1-\kappa)\,(1-p_A)\, p_B$, and that for $\left(y^L, y^L\right)$ is the residual probability. The higher $\kappa \in [0,1]$ is, the stronger is the positive correlation between the siblings' outputs, ranging from statistical independence when $\kappa = 0$ to perfect correlation (both outputs low) when $\kappa = 1$.

At the beginning of the second period, the siblings observe each other's outputs.[9] The *state* at the outset of period two is the vector $\omega = (y_A, y_B) \in \Omega = \left\{y^L, y^H\right\}^2$. Having observed the state ω, both siblings simultaneously choose whether to make a transfer to the other, and if so, how much. After these transfers have been made, the *disposable income*, or *consumption*, of each sibling therefore equals his or her output plus any transfer received from the other sibling minus any transfer given.

In this two-stage game, a pure strategy for player $i \in \{A, B\}$ is a pair $s_i = (p_i, \tau_i)$, where $p_i \in (0,1)$ is i's chosen success probability and $\tau_i : \Omega \to \left[0, y^H\right]$ a function that specifies what transfer, if any, to give in each state ω. Each strategy profile $\mathbf{s} = (s_A, s_B)$ determines the *total utility* to each sibling $i = A, B$ in each state ω:

$$U_i\left(\mathbf{s},\omega\right) = V_i\left(\mathbf{s},\omega\right) + \alpha_i V_j\left(\mathbf{s},\omega\right), \tag{4}$$

where $j \neq i$. Here V_i is sibling i's *material utility,*

$$V_i\left(\mathbf{s},\omega\right) = u(y_i - \tau_i(\omega) + \tau_j(\omega)) - \psi\left(p_i\right)$$

and $\alpha_i \in [0,1]$ represents i's degree of *altruism* of i towards his or her sibling.[10] An individual i with $\alpha_i = 0$ will be called *selfish* and an individual with $\alpha_i = 1$ *fully altruistic.* We solve this two-stage game G by backward induction. Since all four states ω are reached with positive probability under any strategy profile, all Nash equilibria are also sequential equilibria.

3 Equilibrium

In each state $\omega \in \Omega$ at the beginning of the second stage, each sibling i wants to make a transfer to the other if and only if his own marginal material utility from consumption is lower

[9] As will be seen later, our results are unchanged if the siblings also observe each other's efforts.

[10] For $\alpha_i \alpha_j < 1$, Equation (4) can be shown to be equivalent with U_i being proportional to $V_i\left(\mathbf{s},\omega\right) + \alpha_i U_j$ for $i = A, B$, and $j \neq i$. Hence, for such parameter combinations, the current formulation is consistent with "pure" (or "non-paternalistic") altruism; see Lindbeck and Weibull (1988).

than his sibling's, when the latter is weighted by i's degree of altruism. In order to make his transfer decision, individual i also has to figure out whether the sibling is simultaneously planning to give a transfer to him. All that matters to each sibling is the *net* transfer to the other. It is straightforward to prove that, except for the case when both individuals are fully altruistic, in equilibrium at most one sibling makes a transfer, and this transfer is uniquely determined. Should both siblings be fully altruistic ($\alpha_A = \alpha_B = 1$), the transfers are not uniquely determined, but the resulting allocation is uniquely determined. For each state $\omega \in \Omega$, let $G(\omega)$ be the continuation game from the beginning of stage two on, a two-player simultaneous-move game in which each player's strategy is his or her transfer to the other player.

Proposition 1 *For each $\omega \in \Omega$, there exists at least one Nash equilibrium of $G(\omega)$. If $\alpha_A \alpha_B < 1$, then this equilibrium is unique and at most one sibling makes a transfer. A transfer is never made from a poorer to a richer sibling, and the size of the transfer does not depend on the poorer sibling's degree of altruism. If $\alpha_A = \alpha_B = 1$, then there is a continuum of Nash equilibria, all resulting in equal sharing of the total output.*

(Proof in the Appendix.) Let us spell this out in some detail. A positive equilibrium transfer is hence made by a "rich" sibling — a sibling with the high output Y — to a "poor" sibling — a sibling with the low output λY. Let $t(\alpha)$ denote the transfer that a rich sibling with altruism α gives in equilibrium to his or her poor sibling (whose degree of altruism then does not matter). It follows from our assumptions that the transfer given is positive if and only if the rich sibling is sufficiently altruistic in the sense that $\alpha u'(\lambda Y) > u'(Y)$, or, equivalently, if and only if $\alpha > \hat{\alpha}(\lambda)$, where

$$\hat{\alpha}(\lambda) := u'(Y)/u'(\lambda Y) \in (0,1). \tag{5}$$

For each $\alpha > \hat{\alpha}(\lambda)$, the transfer $t \in (0, Y)$ is uniquely determined by the first-order condition

$$u'(Y - t) = \alpha u'(\lambda Y + t). \tag{6}$$

In sum: the transfer $T(\alpha)$ that a rich sibling with altruism $\alpha \in [0,1]$ makes to his or her poor sibling is

$$T(\alpha) = \max\{t, 0\}, \tag{7}$$

where t is defined by (6).

We note that the *equilibrium transfer function* $T : [0,1] \to [0,Y]$ is continuous, positive if $\alpha > \hat{\alpha}(\lambda)$, and zero otherwise. Moreover, T is differentiable for all $\alpha \neq \hat{\alpha}(\lambda)$, with

$$T'(\alpha) = -\frac{u'(\lambda Y + t)}{u''(Y - t) + \alpha u''(\lambda Y + t)} > 0 \tag{8}$$

for all $\alpha > \hat{\alpha}(\lambda)$. Hence, as one would expect, a rich sibling's transfer to his or her poor sibling is strictly increasing in the rich sibling's degree of altruism, for all degrees of altruism above its critical lower bound for a transfer to occur, $\hat{\alpha}(\lambda)$.

The following simple observations turn out to be useful for the subsequent analysis. First, a rich sibling with altruism $\alpha \in (\hat{\alpha}(\lambda), 1)$ always remains richer than his or her poor sibling also after the transfer:

$$c^H = Y - T(\alpha) > \lambda Y + T(\alpha) = c^L.$$

When $\alpha = 1$, total output is shared equally: $Y - T(\alpha) = \lambda Y + T(\alpha)$ for all $\lambda < 1$ and $Y > 0$.

Secondly, for a given level of altruism $\alpha > \hat{\alpha}(\lambda)$ and high-output level $Y > 0$, the equilibrium transfer is increasing in output variability: the higher λ is (and therefore, the higher the low output λY is), the smaller is $T(\alpha)$. However, an increase in λ (lowered variability) is not fully offset by the decrease in the transfer: it leads to strictly higher consumption levels for both siblings in the two states in which one sibling is rich and the other poor. Formally:[11]

Proposition 2 *: Both $c^H = Y - T(\alpha)$ and $c^L = \lambda Y + T(\alpha)$ are increasing in λ.*

Remark 1 *It is easily verified that the equilibrium transfers would have been the same, had the siblings observed each others' effort. This follows from the assumed additive separability of material utility, see equation (2).*

Turning to the first period, in which the siblings simultaneously choose their individual success-probabilities (or, equivalently, efforts), they both anticipate the subsequent transfers

[11] In a model with an altruistic parent and a selfish child, Altonji, Hayashi, and Kotlikoff (1997) showed that an increase in the child's income by \$1 would lead to a decrease of \$1 in the parent's transfer to the child. This result was derived in a model where the parent makes transfers to the child in two subsequent periods, and it hinges on the assumption that the child is liquidity constrained in the first period. Hence, proposition 2 is not in contradiction with their result.

in each of the four possible states in the second period. The *ex ante expected total utility* for each sibling i is thus a function of their choices of success probabilities:

$$\begin{aligned} U_i(p_i, p_j) = {} & (1-\kappa)\, p_A p_B (1+\alpha_i) u\,(Y) \\ & + \left[(1-\kappa)\,(1-p_A)(1-p_B) + \kappa\right](1+\alpha_i) u\,(\lambda Y) \\ & + (1-\kappa)\, p_i (1-p_j)[u\,(Y - T\,(\alpha_i)) + \alpha_i u\,(\lambda Y + T\,(\alpha_i))] \\ & + (1-\kappa)\, p_j (1-p_i)[u\,(\lambda Y + T\,(\alpha_j)) + \alpha_i u\,(Y - T\,(\alpha_j))] \\ & - \psi\,(p_i) - \alpha_i \psi\,(p_j)\,, \end{aligned} \tag{9}$$

for $i = A, B$ and $j \neq i$. The four first terms represent the distinct second-period states: both being rich, both being poor, i rich and j poor, and i poor and j rich (for $i = A, B$ and $j \neq i$). The last two terms represent the two siblings' disutility from effort.

The pair (U_A, U_B) defines the (differentiable) payoff functions in a two-player normal-form game G^* in which a pure strategy for each player i is his or her success probability $p_i \in (0,1)$. Each Nash equilibrium of the reduced-form game G^* induces a Nash equilibrium of G, and *vice versa*. Without loss of generality, we may hence focus on the Nash equilibria of G^*.

A necessary and sufficient condition for a strategy pair $(p_A, p_B) \in (0,1)^2$ to constitute a Nash equilibrium is that it satisfy the following generalization of the optimality condition for the autarky case:[12]

$$\begin{cases} \psi'\,(p_A) = (1-\kappa)\,[u(Y) - u(\lambda Y) + g\,(p_B, \alpha_A, \alpha_B)] \\ \psi'\,(p_B) = (1-\kappa)\,[u(Y) - u(\lambda Y) + g\,(p_A, \alpha_B, \alpha_A)] \end{cases} \tag{10}$$

where, for any $p, \alpha, \beta \in [0,1]$:

$$\begin{aligned} g\,(p, \alpha, \beta) \;=\; & (1-p) \cdot (u\,[Y - T\,(\alpha)] + \alpha u\,[\lambda Y + T\,(\alpha)] - [u\,(Y) + \alpha u\,(\lambda Y)]) \\ & - p \cdot (u\,[\lambda Y + T\,(\beta)] + \alpha u\,[Y - T\,(\beta)] - [u\,(\lambda Y) + \alpha u\,(Y)])\,. \end{aligned} \tag{11}$$

Just as in the autarky case (equation (3)), the equation system (10) requires that the marginal cost of increasing one's success probability (or effort) should equal the expected marginal benefit thereof.

Compared to the autarky case, here the marginal benefit has a composite additional term, given by the expression for $g\,(p, \alpha, \beta)$ given in (11). First, increasing one's success

[12] That these necessary first-order conditions are also sufficient follow by the assumption $\psi'' > 0$, which implies that each sibling's expected total utility is concave in his or her own success probability, p_i.

probability *increases* the probability of being able to help one's sibling, should the sibling become poor. This is the first term in the expression for $g(p, \alpha, \beta)$. Second, increasing one's success probability *decreases* the probability of being helped out by one's sibling, should the sibling become rich. This is the second term.

The right-hand sides in the equation system (10) are decreasing affine functions of the *other* sibling's success probability. Hence, the higher one's sibling's success probability, the weaker is the incentive to increase one's own success probability. This disincentive effect can be decomposed into two components: when i's sibling's success probability (effort) increases, then (a) the probability that i will be put in a position to help, if successful, decreases, and (b) the probability of being helped out if unsuccessful increases. We saw previously that the transfer from a rich to a poor sibling is increasing in the level of altruism of the rich sibling. Will a higher level of altruism therefore lead to lower levels of effort, as suggested by well-known results on moral hazard and insurance?

To answer this question, we first ask how changes in the individual degrees of altruism would affect the equilibrium efforts. Thus, consider an increase in sibling i's altruism: this has only one effect on the transfers, namely, that sibling i would make a larger transfer to his sibling j should i be rich and j poor. Clearly, this should reduce j's incentive to provide effort. But how about sibling i? Sibling i gets to keep less if he is rich and the other poor—intuitively this should have a negative impact on i's effort. However, sibling i now also cares more about j, and this should have a positive impact. It turns out that the latter, positive effect always outweighs the former, negative effect. This claim can be made precise if the Jacobian of the equation system (10) is non-null, a condition that guarantees local uniqueness of the equilibrium in question.

Proposition 3 *Consider a Nash equilibrium (p_A^*, p_B^*) of G^*. If $\alpha_A, \alpha_B < \hat{\alpha}(\lambda)$, then a marginal change of α_A or α_B has no effect on (p_A^*, p_B^*). If (12) holds and $\alpha_i > \hat{\alpha}(\lambda)$, then a marginal increase in α_i causes an increase in p_i^* and a decrease in p_j^* (for $i \in \{A, B\}$ and $j \neq i$).*

$$\psi''(p_A^*)\,\psi''(p_B^*) \neq (1-\kappa)^2 \frac{\partial g(p_A^*, \alpha_B, \alpha_A)}{\partial p_A} \frac{\partial g(p_B^*, \alpha_A, \alpha_B)}{\partial p_B} \tag{12}$$

The intuition behind the proof given in the appendix is straightforward: if some transfer is given with positive probability along the equilibrium path, then an individual's best reply to any success probability that his or her sibling may choose is increasing in the individual's own altruism, *ceteris paribus*. The motive is twofold: first, to increase the chance to have

something to give in case one's sibling obtains the low output, and, secondly, to decrease the risk that one's sibling will need to give a transfer.[13]

In sum, a more altruistic individual not only gives a larger transfer, but also chooses a higher probability of obtaining the high output level. We call this positive effect of altruism the *empathy effect* (from own altruism). By contrast, an individual may choose a lower success probability if the sibling's altruism increases, *ceteris paribus.* This is the well-known *free-riding effect* of others' altruism (here: one's sibling's). If both siblings become more altruistic, will the empathy or free-riding effect dominate? We answer this question for the case of equally altruistic siblings.

4 Equally altruistic siblings

Consider a pair of siblings with the same degree of altruism: $\alpha_A = \alpha_B = \alpha$. The game G^* then has a unique symmetric equilibrium (p^*, p^*), where $p^* \in (0,1)$ solves the following equation, obtained from (10):[14]

$$\begin{aligned} \psi'(p) &= (1-\kappa)\left[u(Y) - u(\lambda Y)\right] \\ &\quad + (1-\kappa)(1-p)\cdot\left(u\left[Y - T(\alpha)\right] + \alpha u\left[\lambda Y + T(\alpha)\right] - \left[u(Y) + \alpha u(\lambda Y)\right]\right) \\ &\quad - (1-\kappa)\,p\cdot\left(u\left[\lambda Y + T(\alpha)\right] + \alpha u\left[Y - T(\alpha)\right] - \left[u(\lambda Y) + \alpha u(Y)\right]\right). \end{aligned} \tag{14}$$

We first consider a parametric example.

[13] Transfers are voluntary, but it is better for a sibling to be in a state in which both siblings receive the high output.

[14] To see that the symmetric equilibrium is unique, note that, by hypothesis, the left-hand side is continuous and increasing from zero to plus infinity, while the right-hand side is a decreasing affine function with positive intercept. The latter property becomes transparent after some algebraic manipulation: equation (14) can be written in the simple form

$$\psi'(p) = (1-\kappa)\left[a - (1+\alpha)\,bp\right], \tag{13}$$

for

$$a = u(Y - T(\alpha)) - u(\lambda Y) + \alpha\left[u(\lambda Y + T(\alpha)) - u(\lambda Y)\right]$$

and

$$b = u(\lambda Y + T(\alpha)) - u(\lambda Y) - \left[u(Y) - u(Y - T(\alpha))\right],$$

where $a, b > 0$. That a is positive follows from our earlier observation that a donor remains richer than the recipient ($u(Y - T(\alpha)) > u(\lambda Y)$). That b is positive follows from the concavity of u, implying that the recipient's material utility increases more from the transfer than the donor's material utility decreases.

4.1 Example

Let the success probability be an exponential function of effort, $\varphi(\theta x) = 1 - e^{-\theta x}$ for $\theta > 0$, and let material utility be log-linear in consumption and effort: $u(y) - v(x) = \ln y - \delta x$ for $\delta > 0$, a parameter that represents the individual's distaste for effort. The expected material utility in autarky, written as a function of the success probability p, is then

$$\ln Y + [(1-\kappa)(1-p) + \kappa] \cdot \ln \lambda + \frac{\delta}{\theta} \ln(1-p). \tag{15}$$

From (6) and (7) we obtain the following expression for the equilibrium transfer from a rich individual with altruism $\alpha \in (0,1)$ to her poor sibling:

$$T(\alpha) = \max\left\{0, \frac{\alpha - \lambda}{1+\alpha}\right\} \cdot Y. \tag{16}$$

We note that this transfer is independent of the parameters θ and δ and that it is increasing in α, from zero for all $\alpha < \hat{\alpha}(\lambda) \equiv \lambda$, towards $(1-\lambda)Y/2$ as $\alpha \to 1$.[15]

The first-order condition (14) for the success probability boils down to

$$(1-\kappa)(1-p)\left[(1-p-\alpha p)\ln\left(\frac{1+\lambda}{1+\alpha}\right) + (\alpha - p - \alpha p)\ln\left(\frac{\alpha(1+\lambda)}{\lambda(1+\alpha)}\right) - \ln\lambda\right] = \frac{\delta}{\theta} \tag{17}$$

For given parameter values, the left-hand side is a polynomial of degree two in p, with paremeters α, κ and λ, while the right-hand side is a constant, the ratio between the distaste for effort and the return to effort. Figure 1 plots its solution, the equilibrium success probability $p^*(\alpha)$ for $\delta/\theta = 0.5$ and $\kappa = 0$, for $\lambda = 0.3$ (the upper curve) and for $\lambda = 0.4$ (the lower curve). When altruism is weak ($\alpha \leq \lambda$), the siblings expect no transfers from each other and therefore choose the autarky effort. As α increases beyond λ, each sibling expects to give (receive) a transfer, should he become rich (poor) and the other sibling poor (rich). We note that the equilibrium success probability (or, equivalently, effort) is non-monotonic in altruism. We also note that in the harsher environment, $\lambda = 0.3$, the equilibrium effort is higher for relatively selfish individuals ($\alpha \leq \lambda$) than for relatively altruistic individuals ($\alpha > \lambda$). Hence, in such environments, altruism has a negative net incentive effect on effort (and hence leads to lower expected incomes). By contrast, in the less harsh environment, $\lambda = 0.4$, very high degrees of altruism (α close to 1) has a positive net incentive effect on effort. This is intuitively plausible, since in less harsh environments (those with higher λ), the autarky effort is low and hence so is the marginal disutility of effort. The free-rider

[15] Recall that the gross transfer is indeterminate at $\alpha = 1$ while the net transfer is the same as when $\alpha \to 1$.

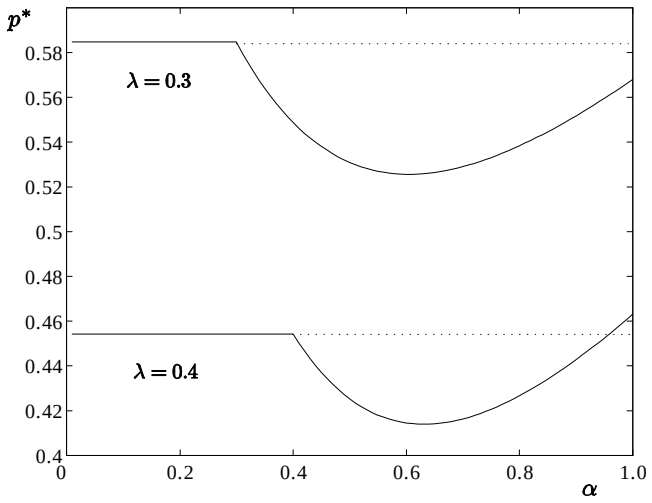

Figure 1: The equilibrium success probability as a function of the common degree of altruism.

effect is therefore weaker than in harsher environments — where the marginal cost of effort in autarky is higher. (The full analysis, leading up to equation (17), also accounts for the empathy effect.)

4.2 Altruism, the external environment and behavior

Does the non-monotonicity of the success probability $p^*(\alpha)$ in the above example hold generally? The answer is affirmative: the free-riding effect dominates at low degrees of altruism while the empathy effect dominates at high degrees of altruism. More precisely, the equilibrium success-probability *decreases* in α when α is at or just above $\hat{\alpha}(\lambda)$ and it *increases* in α when α is near 1.

Proposition 4 *Consider the unique symmetric Nash equilibrium* (p^*, p^*) *of* G^*. *There is an* $\bar{\varepsilon} > 0$ *such that* $p^*(\hat{\alpha}(\lambda)) > p^*(\hat{\alpha}(\lambda) + \varepsilon)$ *and* $p^*(1-\varepsilon) < p^*(1)$ *for all* $\varepsilon \in (0, \bar{\varepsilon})$.

This result is intuitively non-trivial. More altruistic individuals are, by definition, more concerned about the "external effects" that their behavior has on others (here, their sibling), and hence the empathy effect is stronger and free-riding effect weaker on such an individual's behavior when that individual's degree of altruism is increased. However, since here both siblings' degrees of altruism are increased (from the same initial value and by the same marginal amount), the incentive to free-ride on the sibling's increased altruism is also stronger,

so the net effect is *a priori* ambiguous. The above proposition provides a clear-cut result that holds for a wide class of utility functions u and ψ.

Next, let us briefly consider the effects of changes in the exogenous environment on the success probability. Clearly, an increase in the harshness of the environment by way of either an increase in the hazard probability κ, or a decrease in θ, the marginal return to effort, leads to a decrease in the equilibrium success probability.[16] By contrast, an increase in output variability — a decrease in λ — leads to an increase in the equilibrium success probability; a generalization of what we saw in the example in Figure 1.

Proposition 5 *Consider the unique symmetric Nash equilibrium (p^*, p^*) of G^* for a given value of $\alpha \in (0,1)$. Increasing the harshness of the environment affects p^* as follows: it is decreasing in κ, increasing in θ, and decreasing in λ.*

4.3 Altruism and material utility

Still in the special case of a common level of altruism α, we note that a sibling's expected material utility in the unique symmetric equilibrium of G^* may be written as

$$\begin{aligned} V^*(\alpha) &= (1-\kappa)\left[p^*(\alpha)\right]^2 \cdot u(Y) + \left[(1-\kappa)\left[1-p^*(\alpha)\right]^2 + \kappa\right] \cdot u(\lambda Y) \\ &\quad + (1-\kappa)\, p^*(\alpha)\left[1-p^*(\alpha)\right] \cdot \left[u(Y - T(\alpha)) + u(\lambda Y + T(\alpha))\right] \\ &\quad - \psi\left[p^*(\alpha)\right]. \end{aligned}$$

Using this expression it is straightforward to show that the common degree of altruism that leads to the highest *expected material utility* in equilibrium is *full* altruism:

Proposition 6 $V^*(1) \geq V^*(\alpha)$ *for all* $\alpha \in [0,1]$.

The intuition is simple: fully altruistic individuals completely internalize the external effect of their own behavior on their sibling's material utility.[17] Hence, siblings' incentives

[16] This follows from (10) and (11), where ψ is defined by $\psi(p) = v\left(\varphi^{-1}(p)/\theta\right)$. Hence, an increase in θ leads to a downward shift in ψ and ψ'.

[17] Assuming that the siblings are fully altruistic is mathematically equivalent to assuming that they are selfish but make decisions collectively so as to maximize their joint expected material utility, as noted in Arnott and Stiglitz (1991), in a slightly different model.

are perfectly aligned, with each sibling acting like a utilitarian social planner. For lower degrees of altruism, however, their incentives are imperfectly aligned and there is room for some free-riding. From this it is not difficult to show that the expected equilibrium outcome of the interaction between two equally altruistic siblings is *ex ante* Pareto-efficient, in terms of their (imperfectly or perfectly) altruistic preferences, if and only if both siblings are fully altruistic.

Corollary 1 *The symmetric Nash equilibrium* $(p^*(\alpha), p^*(\alpha))$ *of* G^* *is Pareto efficient if and only if* $\alpha = 1$.

At first sight, it may come as a surprise that the outcome is inefficient even when the siblings are purely selfish ($\alpha = 0$). Why does not the independent strife of selfish individuals lead to a Pareto-efficient outcome? The explanation is that both individuals' utility can be increased by keeping their common success probability at its equilibrium level, but having the rich sibling transfer a small amount to the poor sibling, whenever they end up with distinct outputs. Such consumption smoothing across states is beneficial, ex ante, because of the assumed risk aversion (concavity in the utility from consumption). Hence, two selfish siblings would like to write such an (incomplete and mutual) insurance contract, also involving their efforts, had this been possible.

While very high levels of altruism thus are beneficial, it is a non-trivial matter whether moderate levels of altruism are beneficial in terms of the expected material utility. As was shown above, the success probability, and therefore also the expected output, declines as altruism increases from an initially moderate level. It turns out, however, that the expected material utility increases:

Proposition 7 *There is an* $\bar{\varepsilon} > 0$ *such that* $V^*[\hat{\alpha}(\lambda) + \varepsilon] > V^*[\hat{\alpha}(\lambda)]$ *for all* $\varepsilon \in (0, \bar{\varepsilon})$.

5 Evolutionarily robust family ties

A pair of siblings would fare best, in terms of their expected material utility, if they both were fully altruistic. But if sibling altruism is a trait that is inherited from parent to child (where inheritance could be cultural or genetic), is such a high degree of altruism robust against "mutations" towards lower degrees of altruism? As mentioned in the introduction,

"Hamilton's rule" suggests that genetic evolutionary forces would favor a degree of altruism equal to 1/2 between siblings, at least in the absence of strategic elements in their interaction.

In this section we investigate whether "Hamilton's rule" holds up under the strategic sibling interaction modelled here, or if it can be appropriately generalized. In this exploration, we follow and extend somewhat Bergstrom's (1995, 2003) approach. More specifically, suppose that a child (genetically or culturally) inherits either its father's or its mother's degree of sibling altruism ("family values"), with equal probability for both events, and with statistical independence between siblings' altruism draws.[18] Thus, if the father's degree of altruism is α_f and the mother's is $\alpha_m \neq \alpha_f$, then with probability 1/4 two siblings will both have altruism α_f, with the same probability they will both have altruism α_m, and with probability 1/2 one sibling will have altruism α_f and the other α_m. As in Bergstrom's (1995) model, mating is monogamous and mate selection is random.[19]

5.1 Local evolutionary robustness

Consider a sequence of successive, non-overlapping generations, living for one time period each. In each time period, those individuals who survived to the age of reproduction mate in randomly matched pairs. Each pair has exactly two children, and each sibling pair plays the game in section 2.2 once.[20] In the first generation, all individuals have the same degree of sibling altruism $\alpha \in [0, 1]$. Suppose that a "mutation" occurs in the second generation: a small population share of those who are about to reproduce switch to another degree of altruism, $\alpha' \neq \alpha$. Such a switch could be caused by genetic drift, a cultural shift in family values, or it could be due to immigration of individuals with other family values. Random

[18] If the transmission is genetic, this corresponds to the *sexual haploid reproduction* case, where each parent carries one copy of the gene, and the child inherits either the father's or the mother's gene. The human species uses *sexual diploid reproduction*: then each individual has two sets of chromosomes; one set is inherited from the father, and the other from the mother. Whether a gene is expressed or not depends on whether it is recessive (two copies are needed for the gene to be expressed), or dominant (one copy is sufficient for the gene to be expressed). Bergstrom's (2003) analysis of games between relatives shows that the condition for a population carrying the same gene to resist the invasion by a mutant gene in the haploid case is the same as the condition for a population carrying the same recessive gene to resist the invasion by a dominant mutant gene in the diploid case.

[19] See Remark 2 below concerning assortative mating.

[20] Somewhat more generally, each pair may have an even number of children and they interact in pairs.

matching of couples takes place as before and reproduction occurs. We call the "incumbent" degree of altruism α *evolutionarily robust against* α' if a child carrying the incumbent degree of altruism obtains, on average, a higher material utility than a child carrying the mutant degree, for all sufficiently small population shares of the "mutant" degree of altruism, α'. The "incumbent" degree α is *evolutionarily robust* if this holds for every $\alpha' \neq \alpha$.[21]

Let $V(\alpha, \beta)$ denote the expected material utility to an individual with altruism α whose sibling has altruism β. In particular, $V(\alpha, \alpha) \equiv V^*(\alpha)$. As we will presently see, the condition for the above-mentioned incumbent degree of altruism α to be evolutionarily robust against a mutant degree $\alpha' \neq \alpha$ boils down to the following inequality:

$$V^*(\alpha) > \frac{1}{2}\left[V(\alpha', \alpha) + V^*(\alpha')\right]. \tag{18}$$

Formally, we define a degree of sibling altruism $\alpha \in [0, 1]$ to be *evolutionarily robust* if it meets (18) for all $\alpha' \neq \alpha$.[22]

To see that (18) indeed is necessary and sufficient for evolutionary robustness as informally defined above, note that the left-hand side, $V^*(\alpha)$, approximates the expected material utility to a child with the incumbent degree of altruism, α. For if the population share of mutants in the parent generation, $\varepsilon > 0$, is close to zero, then with near certainty both parents of this child are α-altruists, implying that the child's sibling almost surely also is an α-altruist. Likewise, the expression on the right-hand side approximates the expected material utility to a child with the mutant degree of altruism, α'. Because for ε close to zero, such a child almost certainly has exactly one parent with the mutant degree of altruism (the probability that both parents are mutants is an order of magnitude smaller, ε^2, and the probability that none is, is zero). Therefore, with probability close to 1/2 this child's sibling

[21] This notion is similar to that of an evolutionarily stable (pure or mixed) strategy in a finite and symmetric two-player game; a population using such a strategy is robust against a small-scale invasion of any mutant strategy in the sense of earning a higher expected payoff in the post-entry population, see Maynard Smith (1982).

[22] Bergstrom (1995, 2003) derives a condition similar to (18) in a slightly different model, in which each individual is programmed to play a strategy in a symmetric two-player game. Bergstrom shows that for a sexual haploid species, a sufficient condition for a population consisting of x-strategists to be stable against a small invasion of y-strategists is

$$\Pi(x, x) > \frac{1}{2}\Pi(y, x) + \frac{1}{2}\Pi(y, y).$$

where $\Pi(s, s')$ denotes the payoff to strategy s against strategy s'.

has the incumbent degree of altruism, α, and with the complementary probability the sibling has the mutant degree of altruism, α'.

The process by which mutations appear in a population may affect the extent to which the mutant degree of altruism differs from the incumbent degree. In particular, "cultural drift" in values in a society may arguably lead to smaller differences between incumbents and mutants, while immigration from another community or society may sometimes give rise to larger such differences. The relevant evolutionary robustness criterion against "cultural drift" thus is a local version of the above definition. We will call a degree of altruism $\alpha \in [0,1]$ *locally evolutionarily robust* if inequality (18) holds for all $\alpha' \neq \alpha$ near α. Formally:

Definition 1 *A degree of altruism $\alpha \in [0,1]$ is locally evolutionarily robust if (18) holds for all $\alpha' \neq \alpha$ in some neighborhood of α.*

Let us elaborate the notions of evolutionary robustness and local evolutionary robustness a bit. First, note that a degree of altruism α is evolutionarily robust if and only if the right-hand side of (18), viewed as a function of $\alpha' \in [0,1]$, has its unique global maximum at $\alpha' = \alpha$. Second, a degree of altruism α is *locally* evolutionarily robust if and only if the right-hand side of (18), again viewed as a function of $\alpha' \in [0,1]$ has a strict local maximum at $\alpha' = \alpha$. Third, let $\mathcal{A}$ be the degrees of altruism $\alpha \in [0,1]$ such that $V : [0,1]^2 \to \mathbb{R}$ is differentiable at the point (α, α), and define $D : \mathcal{A} \to \mathbb{R}$ by

$$D(\alpha) = V_1(\alpha, \alpha) + \frac{1}{2} V_2(\alpha, \alpha), \tag{19}$$

where V_k is the partial derivative of V with respect to its k'th argument, for $k = 1, 2$.

If the incumbent degree of altruism in a society is $\alpha \in \mathcal{A}$, then $D(\alpha)\, d\alpha$ is the marginal effect of a slight increase in a mutant's degree of altruism, from α to $\alpha + d\alpha$, on its child's expected material utility (achieved in the child's equilibrium play with its sibling) if the child inherits its mutant parent's degree of altruism. The first term is the effect of an increase in the child's own altruism on his or her expected material utility, whereas the second term is the effect of an increase in the child's sibling's altruism, multiplied by one half — the conditional probability that the sibling also is a mutant (in the limit as $\varepsilon \to 0$). We will refer to the function D as the *evolutionary drift function*.

If $D(\alpha) > 0$, then the mutant child, if slightly *more* altruistic than the incumbent population, will outperform the incumbents' children in terms of expected material utility. Likewise, if $D(\alpha) < 0$, then it is instead a mutant child who is slightly *less* altruistic than the

incumbents that will outperform the incumbents' children. Hence, in order for an incumbent degree of altruism $\alpha \in \mathcal{A}$ to be locally evolutionarily robust it is necessary that $D(\alpha) = 0$. Let $int(\mathcal{A}) \subset \mathcal{A}$ be the set of *interior* points in $\mathcal{A}$, that is, degrees of altruism α such that V^* is continuously differentiable at all points (α', α') near (α, α). For such degrees of altruism more can be said:[23]

Proposition 8 *A necessary condition for a degree of altruism $\alpha \in \mathcal{A}$ to be locally evolutionarily robust is $D(\alpha) = 0$. A necessary and sufficient condition for a degree of altruism $\alpha \in int(\mathcal{A})$ to be locally evolutionarily robust is (i)-(iii), where:*

(i) $D(\alpha) = 0$

(ii) $D(\alpha') > 0$ for all nearby $\alpha' < \alpha$

(iii) $D(\alpha') < 0$ for all nearby $\alpha' > \alpha$

In other words: wherever the evolutionary drift function is well-defined, a necessary condition for local evolutionary robustness is that there be no drift, and, that there be upward (downard) drift at slightly lower (higher) altruism levels.

Remark 2 *We have assumed random matching when couples form. Suppose, instead, that mutants have a tendency towards assortative mating: with probability $\sigma \in [0, 1]$ a given mutant will be selective, settle only for a match with another mutant, while with the complementary probability $1 - \sigma$, the mutant will be non-selective and have a random match. For a small population share $\varepsilon > 0$ of mutants, the conditional probability that the sibling to a child with the mutant degree α' of altruism will also have altruism α' is then approximately equal to $(1 - \sigma)/2 + \sigma$ (instead of $1/2$). The evolutionary robustness condition (18) then generalizes to*

$$V^*(\alpha) > \frac{1}{2}\left[(1-\sigma)\, V(\alpha', \alpha) + (1+\sigma)\, V^*(\alpha')\right], \tag{20}$$

and the drift function D becomes

$$D(\alpha) = V_1(\alpha, \alpha) + \frac{1}{2}(1+\sigma)\, V_2(\alpha, \alpha). \tag{21}$$

This boils down to (19) in the limit case of fully random matching and gives $D(\alpha) = V_1(\alpha, \alpha) + V_2(\alpha, \alpha)$ in the case of fully assortative matching.

[23] This follows from the fact that local evolutionary robustness is equivalent with local strict maximization of the right-hand side of (18).

5.2 Application to the present sibling interaction

When applied to the sibling interaction analyzed in sections 2-4 above, we first note that the expected equilibrium material utility to an individual with altruism α and with a sibling with altruism β is

$$\begin{aligned} V(\alpha,\beta) &= (1-\kappa)\,p^*(\alpha,\beta)\,p^*(\beta,\alpha)\,u(Y) \\ &\quad + [(1-\kappa)\,[1-p^*(\alpha,\beta)]\,[1-p^*(\beta,\alpha)]+\kappa]\,u(\lambda Y) \\ &\quad + (1-\kappa)\,p^*(\alpha,\beta)\,[1-p^*(\beta,\alpha)]\,u[Y-T(\alpha)] \\ &\quad + (1-\kappa)\,[1-p^*(\alpha,\beta)]\,p^*(\beta,\alpha)\,u[\lambda Y+T(\beta)]-\psi[p^*(\alpha,\beta)], \end{aligned} \tag{22}$$

where $p^*:[0,1]^2\to(0,1)$ is a function that to each pair of sibling altruism levels, (α,β), associates the equilibrium success probability of the α-altruist.[24] Thus, if an individual has altruism α and his or her sibling has altruism β, then $p^*(\alpha,\beta)$ is the individual's own success probability and $p^*(\beta,\alpha)$ that of the sibling. Such a pair of success probabilities necessarily satisfy the system of equations (10). It follows from (22) that the set $\mathcal{A}$, i.e., the degrees of altruism $\alpha\in[0,1]$ such that V is differentiable at the point (α,α), consists of those degrees of altruism $\alpha\in[0,1]$ that are such that p^* is differentiable at (α,α), and has partial derivatives, $p_1^*(\alpha,\alpha)$ and $p_2^*(\alpha,\alpha)$, with respect to the first and second argument of the function p^*.[25] Recall from proposition 3 that $p_1^*(\alpha,\beta)>0$ and $p_2^*(\alpha,\beta)<0$ whenever $\alpha,\beta>\hat{\alpha}(\lambda)$. Straight-forward calculations based on (22) and the envelope theorem lead to:

Proposition 9 *For any $\alpha\in int(\mathcal{A})$:*

$$\begin{aligned} D(\alpha) &= (1-\kappa)\,p^*(\alpha,\alpha)\,(1-p^*(\alpha,\alpha))\cdot F(\alpha) \\ &\quad + (1-\kappa)\,[(1/2-\alpha)\,p_1^*(\alpha,\alpha)+(1-\alpha/2)\,p_2^*(\alpha,\alpha)]\cdot G(\alpha) \end{aligned} \tag{23}$$

where

$$F(\alpha)=\left[\frac{1}{2}u'[\lambda Y+T(\alpha)]-u'[Y-T(\alpha)]\right]T'(\alpha)$$

[24] We restrict attention to cases in which there is a unique equilibrium. Uniqueness holds, for instance, in the parametric example in Section 3.1 (for details, see Alger and Weibull, 2007). The uniqueness assumption will, in fact, be used only when α and β are (infinitesimally) close to each other.

[25] See, e.g., Theorem 39.6 in Bartle (1976). A sufficient condition for the differentiability of V,at a point $(\alpha,\alpha)\in(0,1)^2$, is that both partial derivatives, p_1^* and p_2^*, exist and are continuous on a neighborhood of (α,α) (see, e.g., Theorem 39.9 in Bartle, 1976).

and

$$G(\alpha) = p^*(\alpha, \alpha) \cdot (u(Y) - u[Y - T(\alpha)]) + [1 - p^*(\alpha, \alpha)] \cdot (u[\lambda Y + T(\alpha)] - u(\lambda Y)).$$

We are now in a position to derive a number of results. These results turn on whether or not $\hat{\alpha}(\lambda) < 1/2$, that is, whether or not the critical degree of altruism for a transfer to occur is lower than Wright's coefficient of relationship between the siblings. Write

$$\hat{\lambda} = \frac{1}{Y}(u')^{-1}[2u'(Y)]. \tag{24}$$

Then $\hat{\alpha}(\lambda) < 1/2$ if and only if $\lambda < \hat{\lambda}$.

We will say that the environment is *gentle* if $\lambda > \hat{\lambda}$**.** In such an environment, the marginal utility at the low output is so close to the marginal utility at the high output level that siblings with altruism $\alpha = 1/2$ do not give any transfers to each other. Hence, their efforts are the same as in autarky. It follows that no degree of altruism $\alpha \leq 1/2$ is evolutionarily locally robust in gentle climates, since a mutant sibling with altruism α' near α does not give any transfer either, and hence it obtains the same expected material utility as a sibling with the incumbent degree of altruism, α.[26]

A more interesting, and arguably empirically more relevant case is when $\lambda < \hat{\lambda}$. In such volatile environments, siblings with altruism $\alpha \geq 1/2$ give voluntary transfers to each other in states when one is rich and the other poor. In the light of Hamilton's rule (Hamilton, 1964a), one might expect $\alpha = 1/2$ to then be the robust degree of kinship altruism. However, in the strategic interaction between siblings studied here, only lower degrees of altruism can be evolutionarily robust:

Proposition 10 *Suppose that* $\lambda < \hat{\lambda}$*. If* $\alpha \in int(\mathcal{A})$ *is locally evolutionarily robust, then* $\hat{\alpha}(\lambda) < \alpha < 1/2$*.*

This result is due to the "strategic externality" that one sibling's altruism exerts on the other's choice of effort: each sibling optimally adjusts its productive effort not only to the exogenous environment but also to the anticipated transfer from the other sibling. To see this, suppose that both siblings' success probabilities were fixed, at some exogenously given level. What levels of sibling altruism α would then be evolutionarily robust? Would Hamilton's rule apply? An application of proposition 8 provides the answer:

[26] In sufficiently gentle environments, mutants who are more altruistic than the incumbents and give transfers fare worse than the incumbents: $D(\alpha') < 0$ for all $\alpha' > \alpha \geq \hat{\alpha}(\lambda)$.

Corollary 2 *Suppose that $\lambda < \hat{\lambda}$ and that efforts are exogenously fixed and equal. Then the unique evolutionarily robust degree of sibling altruism is $\alpha = 1/2$.*

However, in the present model, the success probabilities are endogenous — they are choosen by each sibling, and this choice depends, in general, on the siblings' degrees of altruism, as well as on the exogenous environment $(\lambda, \theta, \kappa)$, where λ is the ratio of the low to the high output, θ the marginal return to effort, and κ the probability of a common negative shock. Hence, which degrees of sibling altruism are locally evolutionarily robust, if any, may depend on the environment. Given the analytical complexity of analyses of this and related questions, we resort to numerical simulations of the example in Section 4.1.

5.3 The external environment and altruism

Here we use the parametric example in Section 4.1 to explore how the environment may affect the evolutionary robustness of different degrees of altruism, and thereby indirectly also effort, income, and material welfare. We note that, logarithmic utility from consumption implies that $\hat{\lambda} = 1/2$ (see equation (24)). In order to keep the number of parameters down, we henceforth set $\delta = 1$.

Figure 2 shows the graph of the evolutionary drift function D, with the common degree of altruism, α, on the horizontal axis, for $\kappa = 0$ and $\theta = 2$. The two graphs correspond to $\lambda = 0.2$ and $\lambda = 0.3$, respectively, where the first represents a harsher environment than the second. Each curve has a discontinuity at its λ-value (recall that $\hat{\alpha}(\lambda) = \lambda$). We see that the evolutionary drift, $D(\alpha)$, is zero for all $\alpha < \hat{\alpha}(\lambda)$. At $\hat{\alpha}(\lambda) < \alpha < 1/2$, $D(\alpha)$ jumps up to a positive value, from which it declines continuously from positive to negative, as α increases towards unity. According to Proposition 8, the intersection of the downsloping curve and the horziontal axis gives the unique locally evolutionary robust degree of sibling altruism. At lower (higher) degrees of sibling altruism there is upward (downward) evolutionary drift. We note that the evolutionarily robust degree of altruism is lower in the harsher environment.

A qualitatively similar effect is found when output variability λ and the return to effort θ are held fixed, and one instead varies the probability of a common negative shock, κ. Figure 3 again shows the graph evolutionary drift function, but now for two different values of κ, where $\kappa = 0.5$ represents a harsher environment than $\kappa = 0$ (in both cases $\lambda = 0.2$ and $\theta = 2$). Although the evolutionary drift function is (pointwise) non-monotonic in κ, we see that, again, the unique evolutionarily robust degree of altruism is lower in the harsher

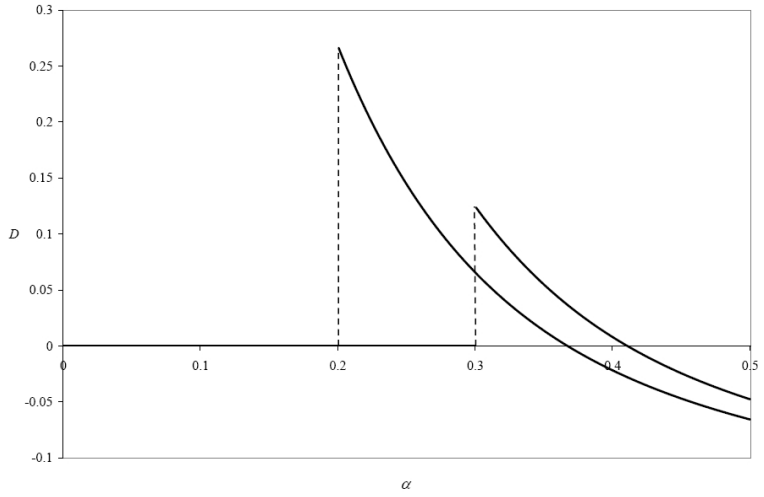

Figure 2: The evolutionary drift $D(\alpha)$ for different output variabilities: $\lambda = 0.2$ (left curve) and $\lambda = 0.3$ (right curve).

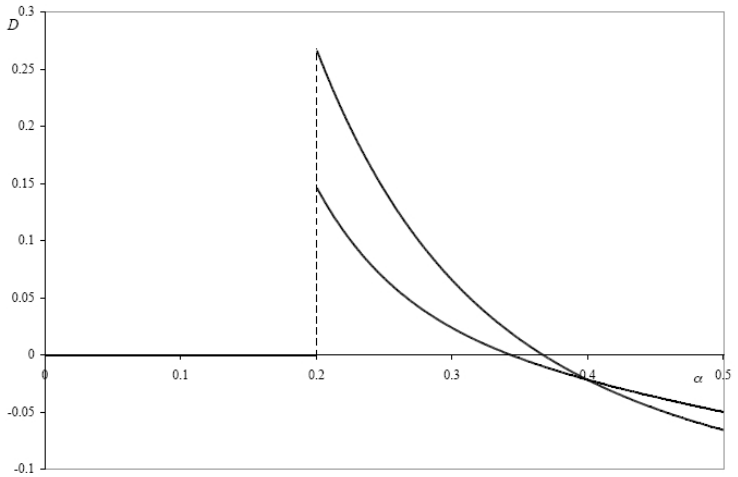

Figure 3: The evolutionary drift $D(\alpha)$ for different probabilities of the exogenous shock: $\kappa = 0$, and $\kappa = 0.5$ (the steeper curve).

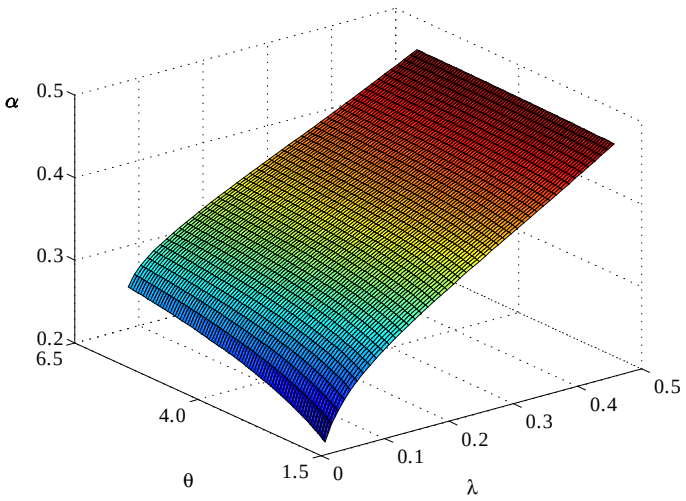

Figure 4: The evolutionarily robust degree of altruism as a function of λ and θ.

environment.

We find a similar effect when output variability λ and the probability κ of a common negative shock are held fixed: a harsher climate in the form of a lower return to effort θ then leads to a lower robust degree of altruism (the figure is omitted).

Figure 4 shows the robust degree of altruism as a function of the environmental parameters λ and θ, with κ held constant (at zero). We see again how the evolutionary forces, as modelled here, select for lower degrees of altruism in harsher environments. This observation might, at first sight, appear counter-intuitive, since risk sharing would seem to have a larger survival value in harsher environments. While this may be true, it may also be that very altruistic populations are more vulnerable to the invasion by slightly less altruistic mutants the harsher is the environment. To see this, consider a relatively altruistic individual who has a more selfish sibling. The altruistic individual suffers doubly from the selfishness of his or her sibling: the selfish sibling both makes a lower effort (Proposition 3) and gives a lower transfer if need be. The altruistic individual is thus more likely to have to help his sibling out, is less likely to be helped out, and receives a lower transfer upon being helped out, than if his sibling had been like him. In harsher environments, both siblings make higher efforts (Proposition 5). Hence, a high degree of altruism may be more vulnerable to selfish mutants in harsher environments.

In sum: our numerical simulations suggest that modest degrees of family altruism will

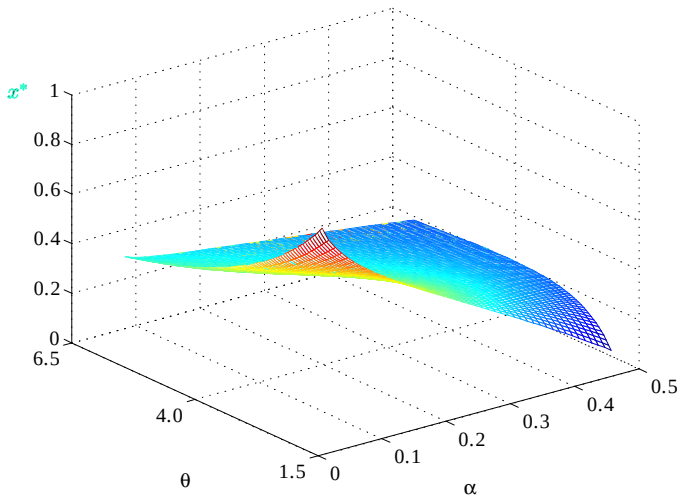

Figure 5: The equilibrium effort x^* as a function of environment $(\lambda, \theta, 0)$ and for robust altruism levels.

prevail in most environments (between 0.2 and 0.5), with higher degrees of family altruism in milder than in harsher climates. In this sense, Darwin lends theoretical support to Weber, in so far as Protestantism is more prevalent in harsher climates and Catholicism in milder climates: evolutionary forces seem to select stronger family ties in milder climates, such as in southern (and mostly Catholic) Europe, than in harsher climates, such as in northern (and mostly Protestant) Europe.

Based on these simulations, we have calculated the equilibrium effort and income as indirect functions of the environment $(\lambda, \theta, \kappa)$, by letting the degree of sibling altruism adapt to its unique evolutionarily robust value in each environment. Figure 5 shows effort, x^*, as such an indirect function of the environment $(\lambda, \theta, \kappa)$, with $\kappa = 0$. For a given value of θ, siblings (with the corresponding evolutionarily robust degree of altruism) exert more work effort in environments with higher output variability (lower λ). In harsher environments in this sense, their family ties are weaker and they work harder. For an outside observer, it is thus as if those who live in milder climates are lazier than those who live in harsher climates, while in all these simulations all individuals actually have identical preferences concerning effort (we have set the distaste for effort, δ, equal to one in all simulations). Max Weber (1904-1905) argued that the "Protestant work ethic" was a key element behind the development of capitalism in northwestern Europe and the United States. Our results suggest that such a work ethic may actually just be a social codification of attitudes that

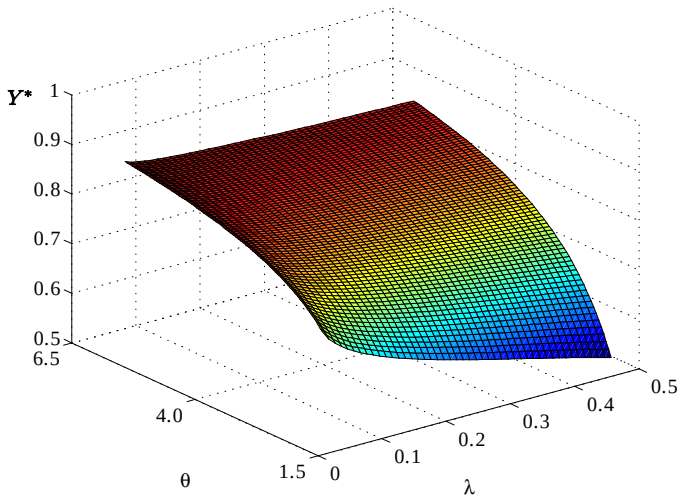

Figure 6: Equilibrium income Y^* as a function of environment $(\lambda, \theta, 0)$ for robust altruism levels.

"nature" has already selected for individuals living in harsher climates.

The effect of θ, the return to effort, is not as clear-cut: for some values of λ, the equilibrium effort, as an indirect function of the environment, is non-monotonic in θ. This is due to two opposing effects, a sort of substitution effect and a sort of income effect. *Ceteris paribus*, an increase in θ has a positive incentive (substitution) effect, but in the new and slightly milder climate, the robust level of altruism is a bit higher, and this has a disincentive (income) effect on effort; for all values of θ in Figure 5, the equilibrium effort level, given the associated robust degree of altruism (adapted to that climate), is lower than the autarky effort level. Note that the same can be said in terms of *technological innovations* in a fixed environment: increased skill (higher θ, say, by means of new tools) may result in higher or lower effort, once family values have adapted to the change in skills.

The higher effort exerted in harsher environments is not always sufficient to yield higher average incomes. Indeed, when family ties adapt to the environment, the expected income may decrease as the environment becomes harsher, see figure 6. Furthermore, even if the expected income sometimes is higher in harsher environments (with lower λ, say), and people thus are richer, they need not be "happier," their expected material utility may be lower. Figure 7 shows the expected *material utility* when family ties have adapted to the unique robust degree of sibling altruism in each environment. Since, moreover, altruism is

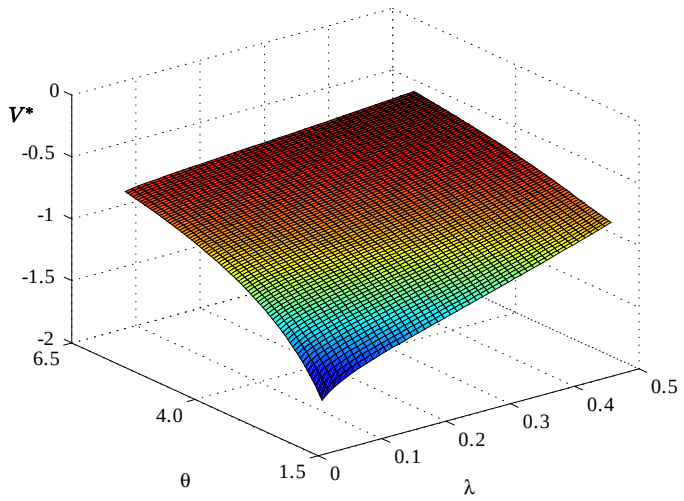

Figure 7: Equilibrium material utility V^* as a function of environment $(\lambda, \theta, 0)$ for robust altruism levels.

lower in harsher environments, this implies that the expected *utility* also is lower in harsher environments.

Our general analysis showed that increased sibling altruism has a non-monotonic effect on effort. This prompts the question whether effort, and thus also the expected individual incomes after transfers, are higher or lower at the evolutionarily robust level of sibling altruism, than if both siblings had been selfish. In the environments in Figures 5 and 6, transfers occur, from a rich sibling to a poor, at the corresponding evolutionarily robust degree of altruism. Figure 8 shows that in all the considered environments, the moral-hazard effect dominates the empathy effect: there is a positive difference between Y^0, the expected income in autarky, and Y^*, the expected income at the evolutionarily robust altruism. Furthermore, the absolute income reduction is higher in harsher climates, despite the lower level of altruism there. However, although altruism (at the evolutionarily robust level in the given environment) thus has a negative effect on expected income, it does increase welfare — the expected material utility. As seen in Figure 9, the difference between V^*, the expected material utility at the evolutionarily robust degree of altruism, and V^0, the expected material utility in autarky, is positive. Moreover, the absolute gain from altruism in material utility is larger in harsher climates, despite the lower level of altruism in such climates.

Remark 3 *Using the expression in (21), it is straightforward to verify that an increase in*

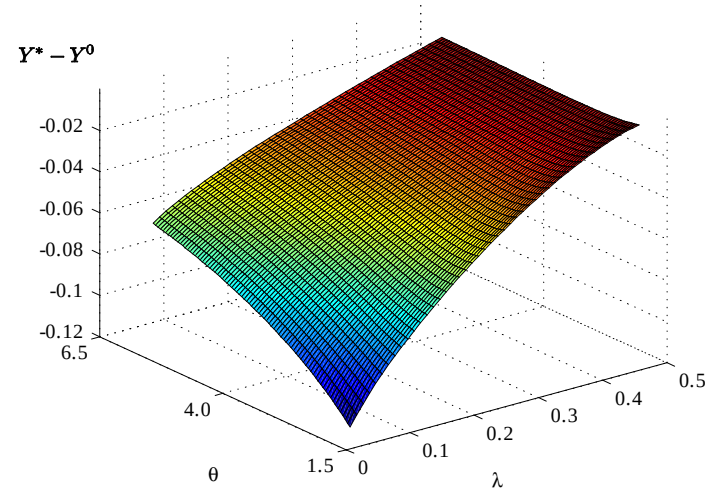

Figure 8: The difference $Y^* - Y^0$ in expected income with and without family ties, as a function of the environment $(\lambda, \theta, 0)$, for robust altruism levels.

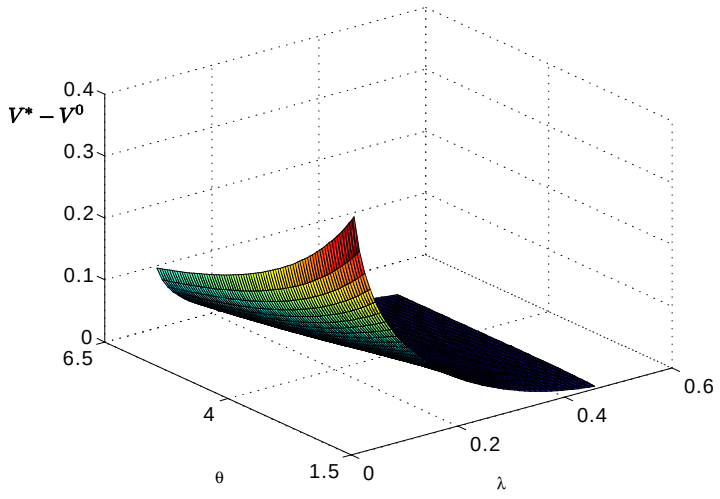

Figure 9: The difference $V^* - V^0$ in expected material utility with and without family ties, as a function of environment $(\lambda, \theta, 0)$, for robust altruism levels.

σ, the degree of assortative mate selection, leads to an upward shift in the drift function D. Hence, assortative mating increases the evolutionarily robust degree of altruism. For example, for the parametric example in Section 4.1 with $(\lambda, \theta, \kappa) = (2, 2, 0)$, the evolutionarily robust degree of sibling altruism is approximately 0.37 under random mating, $\sigma = 0$, and approximately 0.62 when $\sigma = 0.5$. When mutants are more likely to bestow the benefits from their own altruism on other mutants, as is the case if σ is one half rather than zero, the marginal value of a mutation towards a slightly higher level of altruism is higher, and the evolutionarily robust degree of altruism is higher. To us, it is an open question whether or not there is assortative mating under gradual evolutionary drift in family values, so we feel more confident in predictions assuming little or no assortative mating.

6 Evidence on family ties

Our theoretical analysis focuses on the family as a potential source of mutual insurance, and on the mixed incentive effect on individual effort from such potential mutual help within the family. Here we summarize some of the evidence for such phenomena. We also discuss empirical studies by social scientists from different academic disciplines (economics, anthropology, sociology and history), studies suggesting that family ties are weaker in some societies than in others, and that such differences may have predated the industrial revolution. We argue that the evidence is in line with the qualitative predictions of our model, namely, that family ties are stronger in less harsh environments.

First, there is evidence that transfers within the extended family are a source of insurance in countries where formal insurance is less well developed.[27] In a survey on private transfers between households, Cox and Jimenez (1990) conclude that in developing countries 20-90% of households receive (private) transfers from other households (mostly within the same extended family), which can represent up to 20% of the average household income. In the U.S. the corresponding figures are 15% and 1%, respectively. Since the average income of donor households exceeds that of recipient households (Cox, Galasso and Jimenez, 2006), these transfers appear to provide some insurance; see also Cox and Fafchamps (2008). Several other studies, such as Udry (1990), Towsend (1994), Miller and Paulson (2000), and Kurosaki and Fafchamps (2002), confirm the hypothesis that insurance occurs within the extended

[27] In 2003 the total value of insurance premia (life and non-life) as a percent of GDP was 12.48 in the US, 9.85 in France, 1.42 in Turkey, and 1.74 in Mexico (Insurance Statistics Yearbook: 1994-2003, OECD, 2005).

family.

Second, there is some evidence in that the degree of intra-family insurance affects effort. Despite the previously strong emphasis in the theoretical literature on the possible moral-hazard effect of intrafamily altruism (see Laferrère and Wolff, 2006, for a survey), there seems to be a limited number of empirical studies on this topic. Two of those studies suggest that mutual insurance within the extended family induces moral hazard. Using data on farmer output in Mali, Azam and Gubert (2005) establish that recipients of remittances from emigrated relatives in Mali decrease their effort in response to an increase in remittances. Similarly, the analysis of Thai data by Miller and Paulson (1999) reveals that better insurance in the form of remittances leads to more gambling, both among those who are potential remitters, and among those who are likely to receive remittances. By contrast, the findings by Kohler and Hammel (2001) indicate that mutual insurance within the family may have a positive effect on individuals' risk-reducing effort. Using census data for Slavonia from 1698, Kohler and Hammel find that the number of different crops grown by a nuclear family tended to increase as the grain resources available within the extended family network (relative to the household's own land resources, and controlling for physical distance) increased. The authors were expecting the opposite effect, namely that as a result of an increase in the amount and proximity of resources available for risk pooling within the extended family, a household would invest *less* in risk-reducing planting strategies. However, our results provide an explanation for this pattern: when a family expects to help another family out, the expected benefit of the risk-reducing planting strategy is increased. The situation investigated by Azam and Gubert is perhaps closer to a model with one-sided altruism: with remittances, essentially only the emigrant family member is in a position to help out the family that stayed in the home country. Hence, the only effect of family altruism on the latter is the free-riding effect, inducing lower effort. By contrast, Kohler and Hammel studied households living in the same area, suggesting that any household could end up as a donor or a recipient of transfers.

Finally, there is evidence for geographic variations in the strength of family ties. U.S. data collected by Keefe et al (1979) indicates that second and third generation Mexican-American families have stronger kin ties than white Anglo families, even after controlling for variables such as education, occupation and the number of years of residence in the same city. Keefe (1984) further finds that Mexican-Americans (people of Mexican descent but born in the U.S.) attach a larger value than Anglos to the physical presence of family members. Using another data set, Gonzales (1998) shows that Mexican-Americans tend to live closer

to and have more contact with kin than Anglos, even after several generations in the U.S. Her analysis further suggests that both Mexican-Americans and Mexican immigrants are significantly more sympathetic to the idea that parents (adult children) should let their adult children (parents) live with them if in need. This evidence is consistent with our predictions, since on average the climate in Mexico is arguably milder than in the U.S. It also indicates that the strength of family ties perdures for several generations, and that current data may be interpreted as a reflection of the past. Thus, to the extent that the prevailing strength of family ties in the U.S. may be the result of immigration from all over Europe, and that we may expect the climate of the representative immigrant's country of origin to be harsher than in Mexico, these findings indicate that family ties are stronger in milder climates.

Reher (1998) argues that one can measure the strength of a society's family ties by studying the age at which a child leaves his/her parents' home. In 1995, the average age of children living with their parents was 15 in Spain, 18 in Italy, 9 in the UK, 11 in the US, and 13 in Germany (Bentolila and Ichino, 2000). Although these differences may be affected by differences in economic opportunities, availability and cost of housing, and the extent of publicly provided insurance, there is evidence that preferences for cohabitation between parents are children vary among countries. Using U.S. data Rosenzweig and Wolpin (1993) analyzed how the rate of cohabitation between parents and their adult children responded to an exogenous increase in the parents' income: they found that the rate of cohabitation decreased as a result of the increase in the parents' income. Thus, cohabitation between parents and adult children is may be viewed as an inferior good in the U.S. But in other countries it is a normal good: using Italian data Manacorda and Moretti (2006) found that the rate of cohabitation between parents and their adult children *increased* as a result of an exogenous increase in the parents' income. Again, this is consistent with our predictions that family ties are stronger in less harsh climates.

Apart from Weber's suggestion that Protestantism has shattered the "fetters of the sib," the direct evidence from pre-industrial Europe is scarce. However, the little evidence there is appears to be consistent with our theoretical predictions. Hajnal (1982) reports data on servants in northwestern Europe during the 17th-19th centuries; approximately half of all youngsters served outside the parental home at some point, some leaving the parents at the age of 10. Thus, in 17th century England, "the unit of production was the husband and the wife and hired labor, not children" (Macfarlane, 1978). By contrast, in southern and eastern Europe, hired labor was in the same period scarce and children would typically work on the

parents' farm; several related couples and their children would then constitute an extended household. Finally, differences in the legal systems may provide further insights into the strength of family ties. In England, parents had the right to bequeath or sell their assets to anyone. According to Macfarlane (1992), this right may be traced back to the thirteenth century. By contrast, in France the heirs must be given the opportunity to purchase the assets (Macfarlane, 1992).

7 Conclusion

Family ties are stronger in some parts of the world than in others and this may have been so for a long time. It seems that family ties grew weaker in northwestern Europe prior to the industrial revolution, as noted by Weber (1951). This observation prompted us to ask first, how family ties affect economic outcomes, and second, whether evolutionary forces may have shaped family ties differently in different environments. With a preindustrial world in mind, we focused on the family's role as an insurance provider for its members. We modelled a family as a pair of mutually altruistic siblings, who may provide insurance to each other by way of voluntary transfers. In the literature on market insurance and moral hazard the risk-reducing effort is decreasing in the extent of market insurance. By contrast, we found that the risk-reducing effort is *non-monotonic* in the extent of family insurance. This non-monotonicity was seen to be the result of two opposing effects of altruism on effort, the *free-riding effect* and the *empathy effect*. This theoretical finding calls for more empirical studies on the effect of family ties on effort, of which there currently exists only a fairly small number (see Section 6).

In a preindustrial society, most people make their living as subsistence farmers and hunters, the output from which is determined jointly by their efforts and the environment in which they live. In our model we included three environmental factors: the marginal return to effort, and local and global output variability, respectively. For a given level of intra-family altruism, we studied how these environment factors affect individual family members' productive efforts. Our evolutionary analysis showed that neither very weak nor very strong family ties are robust against drift in the strength of family ties. As expected, full altruism — giving equal weight to a sibling's material welfare as to one's own — was seen to be non-robust. If a few individuals in a large population would become slightly less altruistic toward their kin, these "mutants" would do better in terms of material welfare. More surprising,

perhaps, is our finding that this negative result also holds for the degree of altruism dictated by Wright's degree of relationship (1/2 for siblings, 1/8 for cousins etc.). If a few individuals in such a society would become slightly less altruistic towards their kin, then these individuals would in fact do better in terms of material utility.[28] We showed how this deviation from "Hamilton's rule" (Hamilton, 1964a) disappears if one freezes family members' efforts. We developed a generalized version of Hamilton's rule (Proposition) and used this to show that, with endogeneous productive efforts, intermediate degrees of family altruism are robust in many environments. The harsher the environment, the weaker are the family ties. We view this as a Darwinian explanation of Max Weber's finding that Protestantism has shettered the fetters of the sib: perhaps Protestantism (arguably more predominant in harsher climates) was not the primary cause for weaker family ties, but nature. Viewed in this light, Protestantism may have fitted well as a norm-system in harsher climates.

The results reported here are derived under admittedly heroic simplifications. The sibling interaction that we model is very simple and stylized. Its precise mathematical form, is, of course, but one out of many possibilities. However, we believe it is canonical for the interplay between human effort and nature. Of particular interest for future work, would be to develop models that allow for more than two siblings, parent-child interactions, repeated interactions, a richer menu of outcomes, etc. We hope that the concept of local evolutionary robustness, along with the analytical machinery that we have developed here, can be useful for many other studies of related questions, including studies of a richer menu of family relations. Also, extension to other types of transmission mechanisms between and within different generations, including endogeneous social norms, seem highly relevant to our understanding of the relationships between family values and economic develpoment; see Hauk and Saez-Marti (2002), Lindbeck and Nyberg (2006) and Alger and Weibull (2008) for models of other inter- and intra-generational transmission mechanisms.

[28] Likewise, we also show that full selfishness is non-robust; since if a few individuals in such a society would become slightly altruistic they would do better in terms of material utility.

8 Appendix

8.1 Proposition 1

Let $\hat{\tau}_i : \Omega \to \left[0, y^H\right]$ be the function that defines, for every state $\omega \in \Omega$, the transfer that individual i would like to make to his or her sibling *if the latter makes no* transfer to i. Then $\hat{\tau}_i(\omega) = 0$ if $u'(y_i) \geq \alpha_i u'(y_j)$ for $j \neq i$, otherwise the optimal transfer $\hat{\tau}_i(\omega)$ is positive and equates i's marginal material utility to that of his sibling's when weighted by his own (i's) degree of altruism:

$$u'(y_i - \hat{\tau}_i(\omega)) = \alpha_i u'(y_j + \hat{\tau}_i(\omega)).$$

Since the material utility function is separable in consumption and effort, efforts play no role when determining the transfers, only outputs matter. The claims in Proposition 1 follow from the following

Lemma 1 *For each $\omega \in \Omega$, the transfer pair $(\hat{\tau}_A(\omega), \hat{\tau}_B(\omega))$ constitutes a Nash equilibrium of $G(\omega)$. If $\alpha_A \alpha_B < 1$, then this equilibrium is unique. If $\alpha_A = \alpha_B = 1$, then there is a continuum of Nash equilibria, all resulting in equal sharing of the total output.*

Proof: For every state $\omega \in \Omega$, let $\hat{\tau}_i(\omega)$ denote the transfer that individual i would like to make to his or her sibling j *if j makes no transfer to i*. (This transfer is unique by strict convavity of u.) Consider first the two states ω in which $y_A = y_B$. Then $u'(y_i) \geq \alpha_i u'(y_j)$ for $i = A, B$, $i \neq j$, implying that $\hat{\tau}_i(\omega) = 0$, $i = A, B$. Moreover, in such states, $\hat{\tau}_i(\omega) = 0$ is trivially i's unique best response to $\hat{\tau}_j(\omega) = 0$ (for $i = A, B$, $i \neq j$). This establishes the claim in the lemma for all such states ω.

Secondly, consider the two states ω in which $y_A \neq y_B$. Then $\hat{\tau}_i(\omega) = 0$ if $u'(y_i) \geq \alpha_i u'(y_j)$ (for $j \neq i$) and otherwise $\hat{\tau}_i(\omega) \in (0, y_i)$ is the unique solution to the first-order condition

$$u'(y_i - \hat{\tau}_i(\omega)) = \alpha_i u'(y_j + \hat{\tau}_i(\omega)).$$

Suppose that $y_A > y_B$. It follows that then $(\hat{\tau}_A(\omega), 0)$ is a Nash equilibrium of $G(\omega)$. If $\alpha_A \alpha_B = 1$, then this equilibrium is not unique, since also $(\hat{\tau}_A(\omega) + \varepsilon, \varepsilon)$ is a Nash equilibrium for all $\varepsilon \in (0, y_A - \hat{\tau}_A(\omega))$. Likewise, if $y_A < y_B$, then $(0, \hat{\tau}_B(\omega))$ is a Nash equilibrium of $G(\omega)$, and, if $\alpha_A \alpha_B = 1$, so are $(\varepsilon', \hat{\tau}_B(\omega) + \varepsilon')$ for all $\varepsilon' \in (0, y_B - \hat{\tau}_B(\omega))$. This establishes the first and third claim in the lemma.

As for the second claim, suppose that $\alpha_A\alpha_B < 1$ and consider the state ω in which $y_A > y_B$. If $u'(y_A) \geq \alpha_A u'(y_B)$, then $\hat{\tau}_A(\omega) = 0$ and $(0,0)$ is the uniqe Nash equilibrium of $G(\omega)$. Suppose that $u'(y_A) < \alpha_A u'(y_B)$. Suppose that (t_A, t_B) is a Nash equilibrium of $G(\omega)$. If $t_B = 0$, then $t_A = \hat{\tau}_A(\omega)$ is necessary. Clearly, $0 = t_A < t_B$ is incompatible with equilibrium. It remains to show that also $t_A, t_B > 0$ is incompatible with equilibrium. This can be established by way of proof by contradiction. Suppose, thus that (t_A, t_B) is a Nash equilibrium with $t_A, t_B > 0$. Then the following two first-order conditions must both hold:

$$u'(y_A - t_A + t_B) = \alpha_A u'(y_B + t_A - t_B)$$

$$u'(y_B + t_A - t_B) = \alpha_B u'(y_A - t_A + t_B)$$

and hence

$$u'(y_A - t_A + t_B) = \alpha_A\alpha_B u'(y_A - t_A + t_B)$$

implying $\alpha_A\alpha_B = 1$, contradicting our hypothesis that $\alpha_A\alpha_B < 1$. The same reasoning applies to the state ω in which $y_A < y_B$. This establishes the second claim in the lemma.

8.2 Proposition 2

The first-order condition (6) implicitly defines the transfer t as a differentiable function of λ. An application of the implicit function theorem gives

$$\frac{dt}{d\lambda} = -\frac{\alpha u''(\lambda Y + t)}{\alpha u''(\lambda Y + t) + u''(Y - t)} \cdot Y,$$

where, by strict concavity of u, the ratio on the right-hand side is a number in the open unit interval. Hence

$$\frac{d(\lambda Y + t)}{d\lambda} = \left[1 - \frac{\alpha u''(\lambda Y + t)}{\alpha u''(\lambda Y + t) + u''(Y - t)}\right] \cdot Y > 0,$$

and

$$\frac{d(Y - t)}{d\lambda} = \frac{\alpha u''(\lambda Y + t)}{\alpha u''(\lambda Y + t) + u''(Y - t)} \cdot Y > 0.$$

8.3 Proposition 3

First, assume that $\alpha_A, \alpha_B < \hat{\alpha}(\lambda)$. Then $T(\alpha_A) = T(\alpha_B) = 0$, and inspection of (11) shows that the equation system (10) is independent of α_A and α_B. Hence, its solution set is unaffected by a marginal increase in any one or both of these parameters.

Second, assume that condition (12) is met. Then the Jacobian of the equation system (10) is non-null, a condition, which, by the Inversion Theorem (see, e.g., Theorem 41.8 in Bartle, 1976), guarantees local uniqueness of the solution to (3). Suppose that $\alpha_i > \hat{\alpha}(\lambda)$.

Step 1: First, we prove that, for each success probability of the other individual, p_j, individual i's best response is strictly increasing in α_i. From (10) and noting that $(\psi')^{-1}$ is an increasing function, this claim holds if

$$\frac{\partial g(p_j, \alpha_i, \alpha_j)}{\partial \alpha_i} > 0.$$

Using the first-order condition (6) for the transfer $T(\alpha_i)$, we obtain:

$$\frac{\partial g(p_j, \alpha_i, \alpha_j)}{\partial \alpha_i} = (1 - p_j) \cdot [u(\lambda Y + T(\alpha_i)) - u(\lambda Y)] + p_j \cdot [u(Y) - u(Y - T(\alpha_j))]. \quad (25)$$

The expression on the right-hand side is positive, since $\alpha_i > \hat{\alpha}(\lambda)$ implies $T(\alpha_i) > 0$.

Step 2: Secondly, we prove that an increase in α_i does not lead to an increase in p_j. For this claim, it is sufficient to show that

$$\frac{\partial g(p_j, \alpha_i, \alpha_j)}{\partial \alpha_j} \leq 0.$$

Using the first-order condition (6) for the transfer $T(\alpha_j)$, we obtain:

$$\frac{\partial g(p_j, \alpha_i, \alpha_j)}{\partial \alpha_j} = -p_j \cdot (1 - \alpha_i \alpha_j) \cdot u'(\lambda Y + T(\alpha_j)) T'(\alpha_j). \quad (26)$$

The expression on the right-hand side is negative for all $\alpha_j \geq \hat{\alpha}(\lambda)$ and zero for all $\alpha_j < \hat{\alpha}(\lambda)$.

Taken together, the two steps establish the claim in the proposition.

8.4 Proposition 4

Using equation (14), we obtain

$$\begin{aligned} \frac{dp^*}{d\alpha} &= \frac{(1 - p^*)}{K} \cdot [u(\lambda Y + T(\alpha)) - u(\lambda Y)] + \frac{p^*}{K} \cdot [u(Y) - u(Y - T(\alpha))] \\ &\quad - \frac{p^*(1 - \alpha^2)}{K} \cdot u'(\lambda Y + T(\alpha)) T'(\alpha) \end{aligned} \quad (27)$$

where

$$K = \frac{\psi''(p^*)}{(1 - \kappa)} + (1 + \alpha)([u(\lambda Y + T(\alpha)) - u(\lambda Y)] - [u(Y) - u(Y - T(\alpha))]). \quad (28)$$

As $\alpha \downarrow \hat{\alpha}(\lambda)$, at which point p^* is not differentiable, the first two terms in (27) both tend to zero, while the third term is positive. Since it is to be subtracted, we conclude that $dp^*/d\alpha < 0$ for all $\alpha > \hat{\alpha}(\lambda)$ close to $\hat{\alpha}(\lambda)$. Likewise, as $\alpha \uparrow 1$, the third term tends to zero while the first two are positive. Hence, $dp^*/d\alpha > 0$ for all $\alpha < 1$ close to 1.

8.5 Proposition 5

Using equation (14), we obtain

$$\begin{aligned}\frac{dp^*}{d\lambda} &= -\frac{(1-p^*)\,\alpha Y}{K}\cdot\left[u'(\lambda Y) - u'(\lambda Y + t)\right] - \frac{(1-p^*)\,Y}{K}\cdot u'(\lambda Y) \\ &\quad -\frac{p^*\alpha^2 Y}{K}\cdot u'(\lambda Y + t) - \frac{p^*(1-\alpha^2)}{K}\left[Y + \frac{dt}{d\lambda}\right]\cdot u'(\lambda Y + t).\end{aligned}$$

where t is defined in (6) and $K > 0$ in (28). Since u is strictly increasing and concave, and $|dt/d\lambda| < Y$ (see proof of proposition 2), $dp^*/d\lambda < 0$.

8.6 Proposition 6

We proceed in two steps. First, we characterize the socially optimal probability p and transfer t, to be given by the rich to the poor, under a Benthamite social welfare function. Secondly, we verify that these coincide with the equilibrium probabilities p_A^* and p_B^*, and transfers $T(\alpha_A)$ and $T(\alpha_B)$ if and only if $\alpha_A = \alpha_B = 1$.

Step 1: Consider a hypothetical social planner who chooses a probability p and transfer t so as to maximize the sum of the expected material utilities to each individual,

$$\begin{aligned}W(p,t) &= 2\left[(1-\kappa)\,p^2 u(Y) + \left[(1-\kappa)(1-p)^2 + \kappa\right]u(\lambda Y)\right] \\ &\quad +2(1-\kappa)\,p(1-p)\left[u(Y-t) + u(\lambda Y + t)\right] - 2\psi(p).\end{aligned} \tag{29}$$

The necessary first-order condition for an interior solution for p is

$$2pu(Y) - 2(1-p)\,u(\lambda Y) + (1-2p)\left[u(Y-t) + u(\lambda Y + t)\right] = \frac{\psi'(p)}{(1-\kappa)}. \tag{30}$$

Moreover, for any value of p, the value of t that maximizes $W(p,t)$ is such that both individuals end up with the same consumption in all states: $Y - t = \lambda Y + t$.

Step 2: When positive, the equilibrium transfer satisfies (6). Strict concavity of u implies that $Y - T(\alpha) = \lambda Y + T(\alpha)$ if and only if $\alpha = 1$. Hence, $\alpha = 1$ is a necessary condition

for the equilibrium outcome to coincide with the Benthamite optimum. It is also a sufficient condition, since the first-order condition that defines the equilibrium success probability p^*, equation (14), for $T(\alpha) = (1-\lambda)\,Y/2$, coincides with (30), the necessary first-order condition for an interior social optimum, if and only if $\alpha = 1$.

8.7 Corollary 1

Given the symmetry of the unique equilibrium outcome, this is Pareto efficient if and only if it maximizes the sum of both individuals' expected welfare, as defined in equation (4). If each individual chooses a success probability p and gives a transfer t when rich and the other is poor, the mentioned sum is $S(p,t) = (1+\alpha)W(p,t)$, where $W(p,t)$ is defined in (29). For any $\alpha \in [0,1]$, $S(p,t)$ is clearly strictly increasing in $W(p,t)$. But, by proposition 6, the equilibrium expected material utility V^* coincides with the maximum value of $W(p,t)$ if and only if $\alpha = 1$.

8.8 Proposition 7

The claim in the proposition holds if

$$lim_{\alpha \downarrow \hat{\alpha}(\lambda)} \left[\frac{\partial V(\alpha,\beta)}{\partial \alpha} + \frac{\partial V(\alpha,\beta)}{\partial \beta} \right]_{|\beta=\alpha} > 0,$$

where $V(\alpha,\beta)$ is defined in (22). Here $V(\alpha,\beta)$ is the expected equilibrium material utility to an individual whose degree of altruism is α and whose sibling's degree of altruism is β. Likewise, $p^*(\alpha,\beta)$ is the individual's own success probability and $p^*(\beta,\alpha)$ that of the sibling. Such a pair of success probabilities necessarily satisfy the following system of first-order equations, a generalization of (10):

$$\begin{cases} \frac{1}{1-\kappa}\psi'\left[p^*(\alpha,\beta)\right] = \left[1-p^*(\beta,\alpha)\right]\cdot\left(u\left[Y-T(\alpha)\right]+\alpha u\left[\lambda Y+T(\alpha)\right]-\left[u(Y)+\alpha u(\lambda Y)\right]\right) \\ \quad -p^*(\beta,\alpha)\cdot\left(u\left[\lambda Y+T(\beta)\right]+\alpha u\left[Y-T(\beta)\right]-\left[u(\lambda Y)+\alpha u(Y)\right]\right)+\left[u(Y)-u(\lambda Y)\right] \\ \frac{1}{1-\kappa}\psi'\left[p^*(\beta,\alpha)\right] = \left[1-p^*(\alpha,\beta)\right]\cdot\left(u\left[Y-T(\beta)\right]+\beta u\left[\lambda Y+T(\beta)\right]-\left[u(Y)+\beta u(\lambda Y)\right]\right) \\ \quad -p^*(\alpha,\beta)\cdot\left(u\left[\lambda Y+T(\alpha)\right]+\beta u\left[Y-T(\alpha)\right]-\left[u(\lambda Y)+\beta u(Y)\right]\right)+\left[u(Y)-u(\lambda Y)\right]. \end{cases} \tag{31}$$

Letting V_1 and V_2 denote the partial derivatives of V with respect to the first and second argument, respectively, and likewise, using p_1^* and p_2^* to denote the partial derivatives of p^*

with respect to the first and second argument, respectively, we get:

$$\begin{aligned} V_1(\alpha,\beta) &= (1-\kappa)\left[p^*(\beta,\alpha)\,p_1^*(\alpha,\beta)+p^*(\alpha,\beta)\,p_2^*(\beta,\alpha)\right]u(Y) \\ &\quad -(1-\kappa)\left(\left[1-p^*(\beta,\alpha)\right]p_1^*(\alpha,\beta)+\left[1-p^*(\alpha,\beta)\right]p_2^*(\beta,\alpha)\right)u(\lambda Y) \\ &\quad +(1-\kappa)\left(\left[1-p^*(\beta,\alpha)\right]p_1^*(\alpha,\beta)-p^*(\alpha,\beta)\,p_2^*(\beta,\alpha)\right)u\left[Y-T(\alpha)\right] \\ &\quad -(1-\kappa)\left(p^*(\beta,\alpha)\,p_1^*(\alpha,\beta)-\left[1-p^*(\alpha,\beta)\right]p_2^*(\beta,\alpha)\right)u\left[\lambda Y+T(\beta)\right] \\ &\quad -(1-\kappa)\,p^*(\alpha,\beta)\left[1-p^*(\beta,\alpha)\right]u'\left[Y-T(\alpha)\right]T'(\alpha) \\ &\quad -\psi'\left[p^*(\alpha,\beta)\right]p_1^*(\alpha,\beta) \end{aligned} \tag{32}$$

and

$$\begin{aligned} V_2(\alpha,\beta) &= (1-\kappa)\left[p^*(\beta,\alpha)\,p_2^*(\alpha,\beta)+p^*(\alpha,\beta)\,p_1^*(\beta,\alpha)\right]u(Y) \\ &\quad -(1-\kappa)\left(\left[1-p^*(\beta,\alpha)\right]p_2^*(\alpha,\beta)+\left[1-p^*(\alpha,\beta)\right]p_1^*(\beta,\alpha)\right)u(\lambda Y) \\ &\quad +(1-\kappa)\left(\left[1-p^*(\beta,\alpha)\right]p_2^*(\alpha,\beta)-p^*(\alpha,\beta)\,p_1^*(\beta,\alpha)\right)u\left[Y-T(\alpha)\right] \\ &\quad -(1-\kappa)\left(p^*(\beta,\alpha)\,p_2^*(\alpha,\beta)-\left[1-p^*(\alpha,\beta)\right]p_1^*(\beta,\alpha)\right)u\left[\lambda Y+T(\beta)\right] \\ &\quad +(1-\kappa)\,p^*(\beta,\alpha)\left[1-p^*(\alpha,\beta)\right]u'\left[\lambda Y+T(\beta)\right]T'(\beta) \\ &\quad -\psi'\left[p^*(\alpha,\beta)\right]p_2^*(\alpha,\beta). \end{aligned} \tag{33}$$

From the equation system (31) we have:

$$\begin{aligned} \psi'\left[p^*(\alpha,\beta)\right] &= (1-\kappa)\,p^*(\beta,\alpha)(1+\alpha)\,u(Y) \\ &\quad -(1-\kappa)\left[1-p^*(\beta,\alpha)\right](1+\alpha)\,u(\lambda Y) \\ &\quad +(1-\kappa)\left[1-p^*(\beta,\alpha)\right]\left[u\left(Y-T(\alpha)\right)+\alpha u\left(\lambda Y+T(\alpha)\right)\right] \\ &\quad -(1-\kappa)\,p^*(\beta,\alpha)\left[u\left(\lambda Y+T(\beta)\right)+\alpha u\left(Y-T(\beta)\right)\right]. \end{aligned}$$

Using this to replace $\psi'\left[p^*(\alpha,\beta)\right]$ in (32) and (33), and simplifying yields

$$\begin{aligned} V_1(\alpha,\beta) &= (1-\kappa)\left[p^*(\alpha,\beta)\,p_2^*(\beta,\alpha)-\alpha p^*(\beta,\alpha)\,p_1^*(\alpha,\beta)\right]u(Y) \\ &\quad -(1-\kappa)\left(\left[1-p^*(\alpha,\beta)\right]p_2^*(\beta,\alpha)-\alpha\left[1-p^*(\beta,\alpha)\right]p_1^*(\alpha,\beta)\right)u(\lambda Y) \\ &\quad -(1-\kappa)\,p^*(\alpha,\beta)\,p_2^*(\beta,\alpha)\,u\left[Y-T(\alpha)\right] \\ &\quad +(1-\kappa)\,p^*(\beta,\alpha)\,p_1^*(\alpha,\beta)\,\alpha u\left[Y-T(\beta)\right] \\ &\quad +(1-\kappa)\left[1-p^*(\alpha,\beta)\right]p_2^*(\beta,\alpha)\,u\left[\lambda Y+T(\beta)\right] \\ &\quad -(1-\kappa)\left[1-p^*(\beta,\alpha)\right]p_1^*(\alpha,\beta)\,\alpha u\left[\lambda Y+T(\alpha)\right] \\ &\quad -(1-\kappa)\,p^*(\alpha,\beta)\left[1-p^*(\beta,\alpha)\right]u'\left[Y-T(\alpha)\right]T'(\alpha) \end{aligned}$$

and

$$\begin{aligned}
V_2(\alpha,\beta) &= (1-\kappa)\left[p^*(\alpha,\beta)\,p_1^*(\beta,\alpha)-\alpha p^*(\beta,\alpha)\,p_2^*(\alpha,\beta)\right]u(Y)\\
&\quad-(1-\kappa)\left(\left[1-p^*(\alpha,\beta)\right]p_1^*(\beta,\alpha)-\alpha\left[1-p^*(\beta,\alpha)\right]p_2^*(\alpha,\beta)\right)u(\lambda Y)\\
&\quad-(1-\kappa)\,p^*(\alpha,\beta)\,p_1^*(\beta,\alpha)\,u\left[Y-T(\alpha)\right]\\
&\quad+(1-\kappa)\,p^*(\beta,\alpha)\,p_2^*(\alpha,\beta)\,\alpha u\left[Y-T(\beta)\right]\\
&\quad+(1-\kappa)\left[1-p^*(\alpha,\beta)\right]p_1^*(\beta,\alpha)\,u\left[\lambda Y+T(\beta)\right]\\
&\quad-(1-\kappa)\left[1-p^*(\beta,\alpha)\right]p_2^*(\alpha,\beta)\,\alpha u\left[\lambda Y+T(\alpha)\right]\\
&\quad+(1-\kappa)\,p^*(\beta,\alpha)\left[1-p^*(\alpha,\beta)\right]u'\left[\lambda Y+T(\beta)\right]T'(\beta).
\end{aligned}$$

Evaluating these two expressions at $(\alpha,\beta)=(\alpha,\alpha)$, and rearranging terms, we obtain

$$\begin{aligned}
V_1(\alpha,\alpha) &= (1-\kappa)\,p^*(\alpha,\alpha)\left[p_2^*(\alpha,\alpha)-\alpha p_1^*(\alpha,\alpha)\right]\left(u(Y)-u\left[Y-T(\alpha)\right]\right)\\
&\quad+(1-\kappa)\left[1-p^*(\alpha,\alpha)\right]\left[p_2^*(\alpha,\alpha)-\alpha p_1^*(\alpha,\alpha)\right]\left(u\left[\lambda Y+T(\alpha)\right]-u(\lambda Y)\right)\\
&\quad-(1-\kappa)\,p^*(\alpha,\alpha)\left[1-p^*(\alpha,\alpha)\right]u'\left[Y-T(\alpha)\right]T'(\alpha)
\end{aligned}\tag{34}$$

and

$$\begin{aligned}
V_2(\alpha,\alpha) &= (1-\kappa)\,p^*(\alpha,\alpha)\left[p_1^*(\alpha,\alpha)-\alpha p_2^*(\alpha,\alpha)\right]\left(u(Y)-u\left[Y-T(\alpha)\right]\right)\\
&\quad+(1-\kappa)\left[1-p^*(\alpha,\alpha)\right]\left[p_1^*(\alpha,\alpha)-\alpha p_2^*(\alpha,\alpha)\right]\left(u\left[\lambda Y+T(\alpha)\right]-u(\lambda Y)\right)\\
&\quad+(1-\kappa)\,p^*(\alpha,\alpha)\left[1-p^*(\alpha,\alpha)\right]u'\left[\lambda Y+T(\alpha)\right]T'(\alpha).
\end{aligned}\tag{35}$$

Finally, using the first-order equation (6) that defines $T(\alpha)$ for $\alpha>\hat{\alpha}(\lambda)$, and rearranging terms, we get:

$$\begin{aligned}
V_1(\alpha,\alpha)+V_2(\alpha,\alpha) &= (1-\kappa)(1-\alpha)\left[p_1^*(\alpha,\alpha)+p_2^*(\alpha,\alpha)\right]\cdot\\
&\quad\left(p^*(\alpha,\alpha)\left[u(Y)-u\left(Y-T(\alpha)\right)\right]+\left[1-p^*(\alpha,\alpha)\right]\left[u\left(\lambda Y+T(\alpha)\right)-u(\lambda Y)\right]\right)\\
&\quad+(1-\kappa)\,p^*(\alpha,\alpha)\left[1-p^*(\alpha,\alpha)\right](1-\alpha)\,u'\left(\lambda Y+T(\alpha)\right)T'(\alpha).
\end{aligned}$$

This tends to a positive number as α tends to $\hat{\alpha}(\lambda)$ from above, since the first two terms then tend to zero while the last term tends to a positive number.

8.9 Proposition 9

Using the expressions in (34) and (35) for the partial derivatives V_1 and V_2, we obtain from (19):

$$\begin{aligned} D(\alpha) &= \left[V_1(\alpha,\alpha) + \frac{1}{2}V_2(\alpha,\alpha)\right] \\ &= (1-\kappa)\,p^*(\alpha,\alpha)\,[1-p^*(\alpha,\alpha)]\left[\frac{1}{2}u'[\lambda Y + T(\alpha)] - u'[Y - T(\alpha)]\right]T'(\alpha) \\ &\quad + (1-\kappa)\left[\left(\frac{1}{2}-\alpha\right)p_1^*(\alpha,\alpha) + \left(1-\frac{\alpha}{2}\right)p_2^*(\alpha,\alpha)\right]\cdot \\ &\quad \cdot\left[p^*(\alpha,\alpha)\,(u(Y) - u[Y-T(\alpha)]) + [1-p^*(\alpha,\alpha)]\,(u[\lambda Y + T(\alpha)] - u(\lambda Y))\right]. \end{aligned}$$

Recalling that $T(\alpha) = 0$ for all $\alpha < \hat{\alpha}(\lambda)$, that $p_1^* > 0$ and $p_2^* < 0$ for all $\alpha > \hat{\alpha}(\lambda)$, see Proposition 3, and that $p_1^* + p_2^* < 0$ for α slightly above $\hat{\alpha}(\lambda)$, see Proposition 4, it is straightforward to show that the drift function D has the following properties:

1. $D(\alpha) = 0$ for all $\alpha < \hat{\alpha}(\lambda)$.
2. D is continuous at each $\alpha \in \mathcal{A}$.
3. $\lim_{\alpha \downarrow \hat{\alpha}(\lambda)} > 0 \Leftrightarrow \hat{\alpha}(\lambda) < 1/2$, and $\lim_{\alpha \downarrow \hat{\alpha}(\lambda)} < 0 \Leftrightarrow \hat{\alpha}(\lambda) > 1/2$.
4. $D(1/2) < 0 \Leftrightarrow \hat{\alpha}(\lambda) < 1/2$.

8.10 Corollary 2

Assume that $\lambda < \hat{\lambda}$ and that the success probabilties are exogeneously fixed and equal: $p_A = p_B = p \in (0,1)$. For every $\alpha \in int(\mathcal{A})$ we then have

$$D(\alpha) = (1-\kappa)\,p(1-p)\cdot\left[\frac{1}{2}u'[\lambda Y + T(\alpha)] - u'[Y - T(\alpha)]\right]T'(\alpha), \tag{36}$$

where $T'(\alpha) > 0$ for all $\alpha > \hat{\alpha}(\lambda)$. Since $T(\alpha)$ satisfies the first-order condition (6) for all such α, we have, for every $\alpha \in int(\mathcal{A})$ exceeding $\hat{\alpha}(\lambda) < 1/2$:

$$D(\alpha) = (1-\kappa)\,p(1-p)\cdot\left(\frac{1}{2}-\alpha\right)u'[\lambda Y + T(\alpha)]\,T'(\alpha).$$

By Proposition 9, such an α is locally evolutionarily robust if and only if $\alpha = 1/2$. Clearly no $\alpha \leq \hat{\alpha}(\lambda)$ is locally evolutionarily robust, since then α-siblings give not transfers and an α'-sibling does just as well, for any $\alpha' < \alpha$. From (22) we obtain that $\mathcal{A} = \{\alpha \in [0,1] : \alpha \neq \hat{\alpha}(\lambda)\}$. In particular, $(\hat{\alpha}(\lambda), 1) \subset int(\mathcal{A})$. Hence, $\alpha = 1/2$ is the only locally robust degree of altruism.

References

Alger I. and J.W. Weibull (2007): "The Fetters of the Sib: Weber Meets Darwin," Stockholm School of Economics Working Paper 682.

Alger I. and J.W. Weibull (2008): "Family Ties, Incentives and Development: A Model of Coerced Altruism," in Basu, K., and R. Kanbur, eds., (forthcoming) *Arguments for a Better World: Essays in Honor of Amartya Sen, Volume II: Society, Institutions and Development*, Oxford: Oxford University Press.

Alesina A. and P. Giuliano (2007): "The Power of the Family", NBER WP 13051.

Altonji, J.G., F. Hayashi and L. Kotlikoff (1997): "Parental Altruism and Inter Vivos Transfers: Theory and Evidence," *Journal of Political Economy*, 105:1121-66.

Armendáriz B. and J. Morduch (2005): *The Economics of Microfinance.* Cambridge: MIT Press.

Arnott R. and J. Stiglitz (1991): "Moral Hazard and Nonmarket Institutions: Dysfunctional Crowding Out of Peer Monitoring?" *American Economic Review* 81: 179-190.

Azam J.P. and F. Gubert (2005): "Those in Kayes. The Impact of Remittances on their Recipients in Africa", *Revue Economique,* 56:1331-1358.

Banfield E. (1958): *The Moral Basis of a Backward Society.* Chicago: Free Press.

Bartle R. (1976): *The Elements of Real Analysis, Second Edition.* New York: John Wiley and Sons.

Becker G. (1974): "A Theory of Social Interactions", *Journal of Political Economy* 82, 1063-1093.

Bentolila S. and A. Ichino (2000): "Unemployment and Consumption: Are Job Losses Less Painful near the Mediterranean?", CESifo WP No. 372.

Bergstrom, T.C. (1995): "On the Evolution of Altruistic Ethical Rules for Siblings," *American Economic Review,* 85:58-81.

Bergstrom, T.C. (2003): "The Algebra of Assortative Encounters and the Evolution of Cooperation," *International Game Theory Review* 5:211-228.

Bramoullé Y. and R. Kranton (2007): "Risk-Sharing Networks", *Journal of Economic Behavior and Organization,* 64:275-294.

Bruce N. and M. Waldman (1990): "The Rotten-Kid Theorem Meets the Samaritan's

Dilemma," *Quarterly Journal of Economics,* 105:155-165.

Chami, R. (1998): "Private Income Transfers and Market Incentives," *Economica*, 65:557-580.

Chen, Z, and F. Woolley (2001): "A Cournot-Nash Model of Family Decision Making," *Economic Journal*, 111:722-748.

Coate, S. (1995) "Altruism, the Samaritan's Dilemma, and Government Transfer Policy," *American Economic Review,* 85:46-57.

Coate, S. and M. Ravallion (1993) "Reciprocity without Commitment: Characterization and Performance of Informal Insurance Arrangements," *Journal of Development Economics*, 40:1-24.

Cox, D. and M. Fafchamps (2008): "Extended Family and Kinship Networks: Economic Insights and Evolutionary Directions," in T. Schultz T. and J. Strauss, eds., *Handbook of Development Economics, 4.* Amsterdam: North-Holland.

Cox, D., E. Galasso, and E. Jimenez (2006): "Private Transfers in a Cross Section of Developing Countries," CRR WP 2006-2.

Cox, D. and E. Jimenez (1990): "Achieving Social Objectives through Private Transfers: A Review," *World Bank Research Observer,* 5:205-18.

Edgeworth, F.Y. (1881): *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences.* London: Kegan Paul.

Eshel, I., and A. Shaked (2001): "Partnership," *Journal of Theoretical Biology,* 208:457-474.

Foster, A. and M.R. Rosenzweig (2001) "Imperfect Commitment, Altruism and the Family: Evidence from Transfer Behaviour in Low-Income Rural Areas," *Review of Economics and Statistics,* 83: 389-407.

Genicot, G. and D. Ray (2003) "Group Formation in Risk-Sharing Arrangements," *Review of Economic Studies,* 70: 87-113.

Gonzales, A. (1998): *A Comparative Study of Family Values and Structures among Mexican Americans and Non-Hispanic Whites.* Doctoral Dissertation, The University of Texas at Austin.

Hajnal, J. (1982): "Two Kinds of Preindustrial Household Formation," *Population and*

Development Review, 8:449-494.

Haldane, J.B.S. (1955): "Population Genetics", New Biology, 18:34-51.

Hamilton, W.D. (1964a): "The Genetical Evolution of Social Behaviour. I," *Journal of Theoretical Biology*, 7:1-16.

Hamilton, W.D. (1964b): "The Genetical Evolution of Social Behaviour. II," *Journal of Theoretical Biology*, 7:17-52.

Hauk E. and M. Saez-Martí (2002): "On the Cultural Transmission of Corruption," *Journal of Economic Theory* 107, 311-335.

Keefe, S.E (1984): "Real and Ideal Extended Familism Among Mexican Americans and Anglo Americans: On the Meaning of Close Family Ties," *Human Organization,*43:65-70.

Keefe, S.E, A.M. Padilla, and M.L. Carlos (1979): "The Mexican American Extended Family as an Emotional Support System," *Human Organization,*38:144-152.

Knack, S. and P. Keefer (1997): "Does Social Capital Have an Economic Payoff? A Cross-Country Investigation," *Quarterly Journal of Economics,* 112:1251-1288.

Kohler, H.-P. and E.A. Hammel (2001): "On the Role of Families and Kinship Networks in Pre-Industrial Agricultural Societies: An Analysis of the 1698 Slovenian Census," *Journal of Population Economics*, 14:21-49.

Kurosaki, T. and M. Fafchamps (2002): "Insurance Market Efficiency and Crop Choices in Pakistan," *Journal of Development Economics*, 67:419-453.

Kolm, S.-C., and J.M Ythier, eds. (2006): *Handbook of Reciprocity, Giving and Altruism.* Amsterdam: North-Holland.

Laferrère, A. and F.-C. Wolff (2006): "Microeconomic Models of Family Transfers," in Kolm, S.-C., and J.M Ythier, eds., *Handbook of Reciprocity, Giving and Altruism 2.* Amsterdam: North-Holland.

Lindbeck, A. and S. Nyberg (2006): "Raising Children to Work Hard: Altruism, Work Norms and Social Insurance", *Quartely Journal of Economics,* 121: 1473-1503.

Lindbeck, A. and J.W. Weibull (1988): "Altruism and Time Consistency: The Economics of Fait Accompli," *Journal of Political Economy,* 96:1165-1182.

Macfarlane, A. (1978): *The Origins of English Individualism.* Oxford: Basil Blackwell.

Macfarlane, A. (1992): "On Individualism," *Proceedings of the Britsh Academy,* 82:171-

199.

Manacorda, M. and E. Moretti (2006): "Why do Most Italian Youths Live with Their Parents? Intergenerational Transfers and Household Structure," *Journal of the European Economic Assocation*, 4:800-829.

Miller, D. and A.L. Paulson (1999): "Informal Insurance and Moral Hazard: Gambling and Remittances in Thailand," mimeo Princeton and Northwestern.

Montesquieu, C.-L. (1748): *De l'esprit des lois.* Genève: Barrillot & Fils.

Ozment, S. (1975): *The Reformation in the Cities: The Appeal of Protestantism to Sixteenth-Century Germany and Switzerland.* New Haven: Yale University Press.

Ozment, S. (1992): *Protestants: The Birth of a Revolution.* New York: Doubleday.

Putnam, R.D. (1993): *Making Democracy Work. Civic Traditions in Modern Italy.* Princeton: Princeton University Press.

Reher, D.S. (1998): "Family Ties in Western Europe: Persistent Contrasts," *Population and Development Review,* 24:203-234.

Rosenzweig, M.R. and K. Wolpin (1993): "Intergenerational Support and the Life-Cycle Incomes of Young Men and Their Parents: Human Capital Investments, Coresidence, and Intergenerational Financial Transfers," *Journal of Labor Economics,* 11:84-112.

Sober, E., and D.S. Wilson (1998): *Unto Others.* Cambridge: Harvard University Press.

Trivers, R.L. (1971): "The Evolution of Reciprocal Altruism," *Quarterly Review of Biology*, 46:35-57.

Townsend, R. (1994): "Risk and Insurance in Village India," *Econometrica*, 62:539-591.

Udry, C. (1990): "Rural Credit in Northern Nigeria: Credit as Insurance in a Rural Economy," *World Bank Economic Review*, 4:251-269.

Weber, M. (1904-05): "Die protestantische Ethik und der 'Geist' des Kapitalismus," *Archiv für Sozialwissenschaft u. Sozialpolitik*, 20:1-54, 21:1-110.

Weber, M. (1951): *The Religion of China.* Glencoe: The Free Press.

Wright, S. (1922): "Coefficients of Inbreeding and Relationship," *American Naturalist,* 56:330-338.